DE GRUYTER OPEN
RIGA TECHNICAL UNIVERSITY
Transport and Aerospace Engineering
ISSN 2255-9876 (online)
ISSN 2255-968X (print)
December 2016, vol. 3, pp. 71–79
doi: 10.1515/tae-2016-0009
https://www.degruyter.com/view/j/tae

Algorithm of Analytical Bank-And-Climb Gyro Unit Without a Heading Gyro

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Abstract **– An analytical bank-and-climb gyro unit supplies the flight crew with information about aircraft angular movement relative to the local vertical. This article describes the particular principles of unit operation providing a step by step explanation of principles governing the operation of the analytical bank-and-climb gyro unit and provides a kinematic scheme of the unit. The article also gives an insight into the occurrence of errors in the bank-and-climb gyro unit as well as explains their significance.**

Keywords **– Angular movement, bank-and-climb gyro unit, body-fixed system, geographical reference system.**

I. Introduction

An inertial attitude and heading reference system (analytical bank-and-climb gyro unit) is a technical device which supplies the flight crew with information about aircraft angular movement relative to the local vertical (roll and pitch angles) and has no angular movement sensing devices [1].

The analytical bank-and-climb gyro unit consists of a gyro, accelerometers and an analytical unit. The gyro reflects the direction of the Earth's local vertical. In flight, the direction of gyro rotor axis *C* coincides with vertical axis *H* of the geographical reference system (GRS). The accelerometers measure the aircraft's accelerations. The analytical unit computes the angular velocity of gyro rotor axis *C* in inertial space.

II. Principles of Operation

The operating principle of the analytical bank-and-climb gyro unit includes the following fundamental actions:

1) measuring the aircraft's accelerations;
2) computing the angular velocity of the rotor axis *C* of the gyro;
3) correcting the position of the gyro.

In order to determine the analytical principle of bank-and-climb gyro unit operation, the analysis of the gyro is required [2]. The initial situation is as follows: an aircraft with a three-stage gyro is in flight. In the beginning the gyro's rotor is aligned vertically. Therefore, the direction of gyro rotor axis *C* coincides with vertical axis *H* of the GRS. The primary function of the given unit is to keep the gyro's rotor axis *C* aligned with GRS axis *H*.

In contradistinction to GRS vertical axis *H*, the alignment of the gyro rotor in inertial space is constant. Consequently, in order to have gyro rotor axis *C* aligned with GRS axis *H* during flight, the following angular velocity (1) must be applied to axis *C*:

$$\Omega_C = \Omega_{H,} \tag{1}$$

where

Ω_H angular velocity of GRS vertical axis *H* in inertial space;

Ω_C angular velocity of gyro rotor axis *X* in inertial space.

The Earth is rotating in inertial space with angular velocity Ω_Z. Angular velocity Ω_Z of this rotation is then projected onto GRS axes – *NEH* (see Fig. 1).

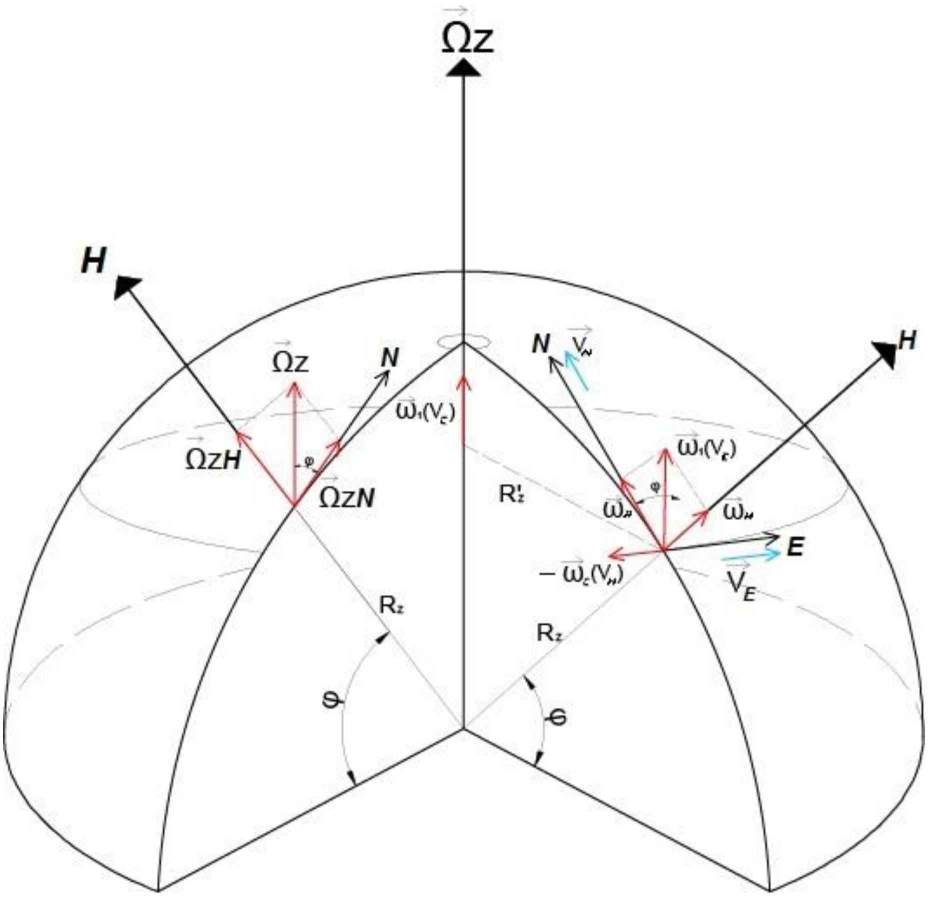

Fig. 1. Fundamental interrelations in the geographical reference system.

The projections (2) of the Earth's rotation angular velocity Ω_Z onto GRS axes (*NEH*) are the following:

$$\begin{cases} \Omega_{ZN} = \Omega_Z \cos\varphi \\ \Omega_{ZH} = \Omega_Z \sin\varphi \end{cases} \tag{2}$$

where

φ geographical latitude angle;

Ω_Z Earth's rotation angular velocity.

The horizontal (North) Ω_{ZN} and vertical Ω_{ZH} components of the Earth's rotation angular velocity appear due to the deviation of GRS vertical axis *H* from the Earth's vertical line. Horizontal (North) component Ω_{ZN} affects the pitch and roll angles, while vertical component Ω_{ZH} affects the yaw angle.

Another reason of *H* rotation in inertial space is the aircraft's movement relative to the Earth's surface. During the flight, the aircraft passes through a variety of points, the local verticals of which have different directions. Therefore, if the aircraft has flight speed *V*, the vector of vertical axis *H* rotates in inertial space. Furthermore, the angular velocity of the vector of vertical axis *H* depends on the value of flight speed *V*. The North component of flight speed V_N results in the appearance of the horizontal component (3) of vertical axis *H* angular velocity ω_E.

$$\omega_E = -\frac{V_N}{R_Z}, \tag{3}$$

where

R_Z Earth's radius;

V_N North component of flight speed.

The East component of flight speed V_E gives origin to the appearance of the vertical component (4) of vertical axis *H* angular velocity $\omega_1(V_E)$.

$$\omega_1(V_E) = \frac{V_E}{R''_Z}, \tag{4}$$

where

$\boldsymbol{R''_Z}$ Projection of Earth radius (5) onto the flight longitude:

$$R''_Z = R_Z cos\varphi. \tag{5}$$

The projections (6), (7) of angular velocity ω_1 of GRS vertical axis *H* onto the *NEH* system are the following:

$$\omega_N = \omega_1 cos\varphi = \frac{V_E}{R''_Z} cos\varphi = \frac{V_E}{R_Z}; \tag{6}$$

$$\omega_H = \omega_1 sin\varphi = \frac{V_E}{R''_Z} sin\varphi = \frac{V_E}{R_Z}\ tg\varphi. \tag{7}$$

The angular velocity $\boldsymbol{\Omega_{Hi}}$ (8) of GRS vertical axis *H* rotation in inertial space (taking into account both causes of this rotation) therefore is:

$$\Omega_{Hi} = \Omega_{Zi} + \omega_i(V_E). \tag{8}$$

The angular velocity $\boldsymbol{\Omega_H}$ of GRS vertical axis *H* consists of two components, which emerge due to different movement processes. The first component $\boldsymbol{\Omega_{Zi}}$ appears because of the Earth's rotation in inertial space. The second component ω_i appears because of the aircraft's movement relative to the Earth's surface.

The components (9), (10) of the rotation velocity $\boldsymbol{\Omega_H}$ of GRS vertical axis *H* are the following:

$$\Omega_{HN} = \Omega_Z cos\varphi + \frac{V_E}{R_Z}; \tag{9}$$

$$\Omega_{HE} = -\frac{V_N}{R_Z}. \tag{10}$$

These mathematical expressions represent the algorithm of analytical bank-and-climb gyro unit operation. In flight, the gyro stabilized platform, which is aligned along North and East axes in the geographical reference system *NEH*, must rotate with the specified velocities in order to keep the platform in the local horizontal plane at all times.

To transform $\boldsymbol{\Omega_{HN}}$ and $\boldsymbol{\Omega_{HE}}$ angular velocities into mathematical expressions, the dependence of Earth's rotation velocity $\boldsymbol{\Omega_Z}$ (11) on the East component of linear velocity $\boldsymbol{V_E}$ should be taken into account:

$$\Omega_Z = \frac{V_E}{R_Z cos\varphi}. \tag{11}$$

Moreover, it should also be noted that Earth's linear velocity (East direction) V_E (12) is an East component of the aircraft's transfer velocity $\boldsymbol{V_{eE}}$:

$$V_E = V_{aE}. \tag{12}$$

Besides, the aircraft's linear velocity $\boldsymbol{V_{e\,a/c}}$ (13) is an East component of the aircraft's relative velocity $\boldsymbol{V_{rE}}$:

$$V_{e\,a/c} = V_{rE}. \tag{13}$$

The sum of the East components of aircraft transfer velocity $\boldsymbol{V_{eE}}$ and relative velocity $\boldsymbol{V_{rE}}$ (14) forms an East component of the aircraft's absolute velocity $\boldsymbol{V_{aE}}$:

$$V_{eE} + V_{rE} = V_{aE}. \quad (14)$$

With account of the above mentioned mathematical expressions, the component of the angular velocity $\boldsymbol{\Omega_H}$ (15) of GRS vertical axis *H* is:

$$\Omega_{HN} = \Omega_Z cos\varphi + \frac{V_E}{R_Z} = \frac{V_E cos\varphi}{R_Z cos\varphi} + \frac{V_{e\,a/c}}{R_Z} = \frac{V_{eE}+V_{rE}}{R_Z} = \frac{V_{aE}}{R_Z}. \quad (15)$$

The North component V_N (17) of the aircraft's linear velocity is a North component of the aircraft's relative velocity V_{rN}:

$$\Omega_{HE} = -\frac{V_N}{R_Z} = -\frac{V_{rN}}{R_Z}. \quad (16)$$

As a result, the following expressions (17), (18) define the analytical operation of the bank-and-climb gyro unit:

$$\Omega_{HN} = \frac{V_{aE}}{R_Z}; \quad (17)$$

$$\Omega_{HE} = -\frac{V_{rN}}{R_Z}, \quad (18)$$

where

$\boldsymbol{\Omega_{HN}}$ North component of the angular velocity of GRS vertical axis *H*;
$\boldsymbol{\Omega_{HE}}$ East components of the angular velocity of GRS vertical axis *H*;
$\boldsymbol{V_{aE}}$ East component of the aircraft's absolute velocity;
$\boldsymbol{V_{rN}}$ North component of the aircraft's relative velocity.

These particular mathematical expressions suggest what should be done in order to design the analytical bank-and-climb gyro unit. Firstly, the East component of the aircraft's absolute flight speed $\boldsymbol{V_{aE}}$ and the North component of the aircraft's relative flight speed $\boldsymbol{V_{rN}}$ should be found. This can be achieved by integrating signals (19), (20) from accelerometers:

$$V_{aE} = \int a_{aksE} dt; \quad (19)$$

$$V_{rN} = \int a_{aksN} dt, \quad (20)$$

where $a_{\mathrm{aks}E}$ and $a_{\mathrm{aks}N}$ are East and North accelerometer signal components accordingly.

The expressions (19), (20) represent the operating algorithm of the analytical bank-and-climb gyro unit [3]–[9].

III. Kinematic Scheme of the Unit

There can be various engineering implementations. Let us consider a relatively simple analytical bank-and-climb gyro unit without the heading stabilization of the platform and with stabilization in the Earth's horizontal plane only [10].

Fig. 2 shows the kinematic scheme of this analytical bank-and-climb gyro unit.

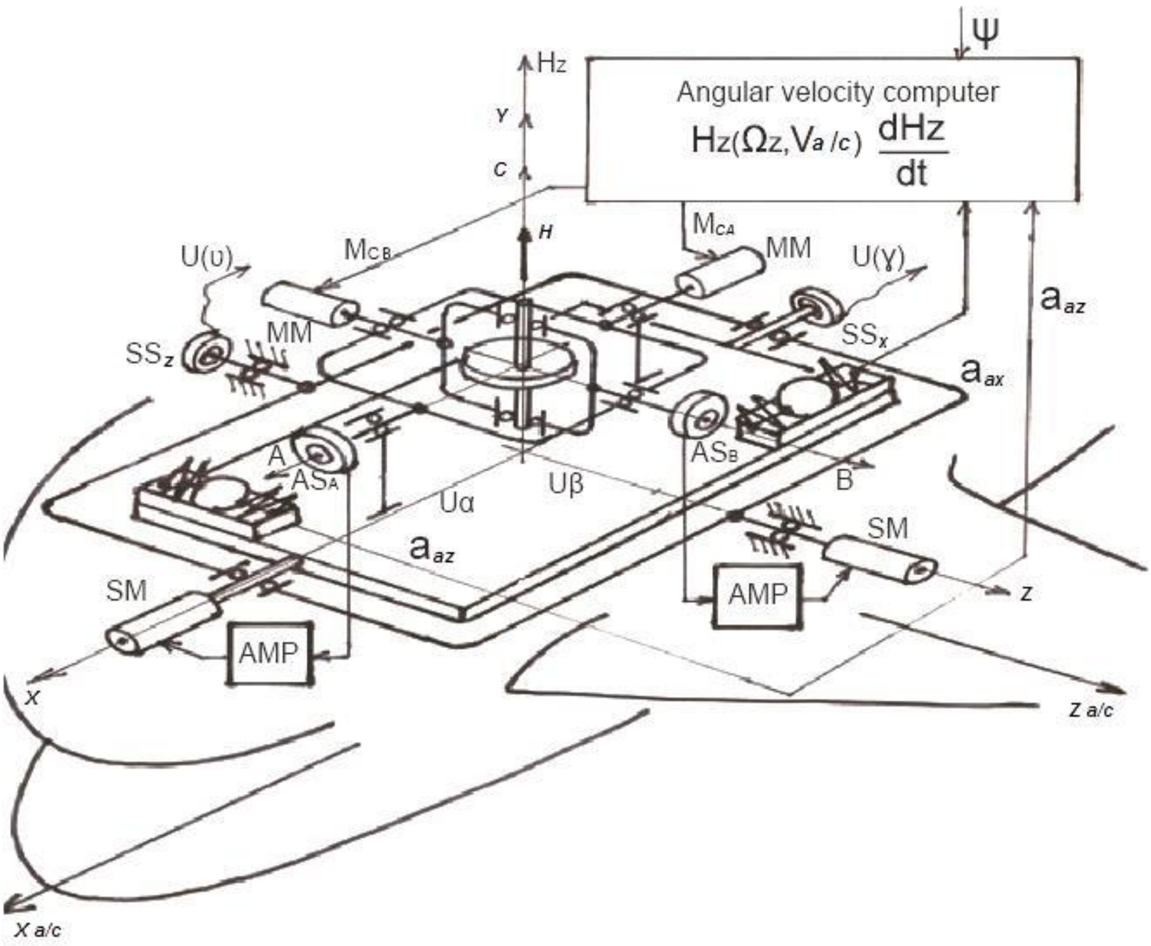

Fig. 2. Kinematic scheme of analytical bank-and-climb gyro unit.

The analytical bank-and-climb gyro unit consists of the following components:

1. A leveling gyroplatform with a plane situated in the local horizon plane during the whole operation of the system;
2. Two accelerometers measuring the aircraft's accelerations along the platform axes in inertial space;
3. An angular velocity computer calculating the angular velocity of the Earth's local vertical;
4. Functional components of the angular velocity computer – moment motors (electric motors), which generate control moments for the gyro;
5. A platform correction circuit consisting of gyro angular sensors, amplifiers and stabilization motors. The platform correction circuit rotates the platform in reliance on gyro signals (the platform is set in such a way that its plane is perpendicular to the gyro rotor) [11].

IV. Basis of the Operating Principle

The principle of operation of the analytical bank-and-climb gyro unit is the following: the accelerometers measure absolute accelerations along the platform axis, as they are mounted on the platform, while the platform is mounted to the aircraft. As a result, the analytical bank-and-climb gyro unit measures absolute accelerations along the axes of body-fixed reference system X and Z – a_{ax} and a_{az}.

However, to compute the angular velocity of the Earth's local vertical in inertial space, information about flight speed components V_E and V_N should be obtained in the GRS [12], [13]. Therefore, acceleration components a_{ax} and a_{az} should be transferred from the body-fixed reference system XYZ to the geographical reference system NEH (a_{aN} and a_{aE}). The transfer can be completed if the transfer angle between both reference systems is known. The body-fixed reference system and geographical reference system are shown in Fig. 3. For instance, let us assume that the body-fixed reference system is rotated relative to the geographical reference system for an angle of ψ.

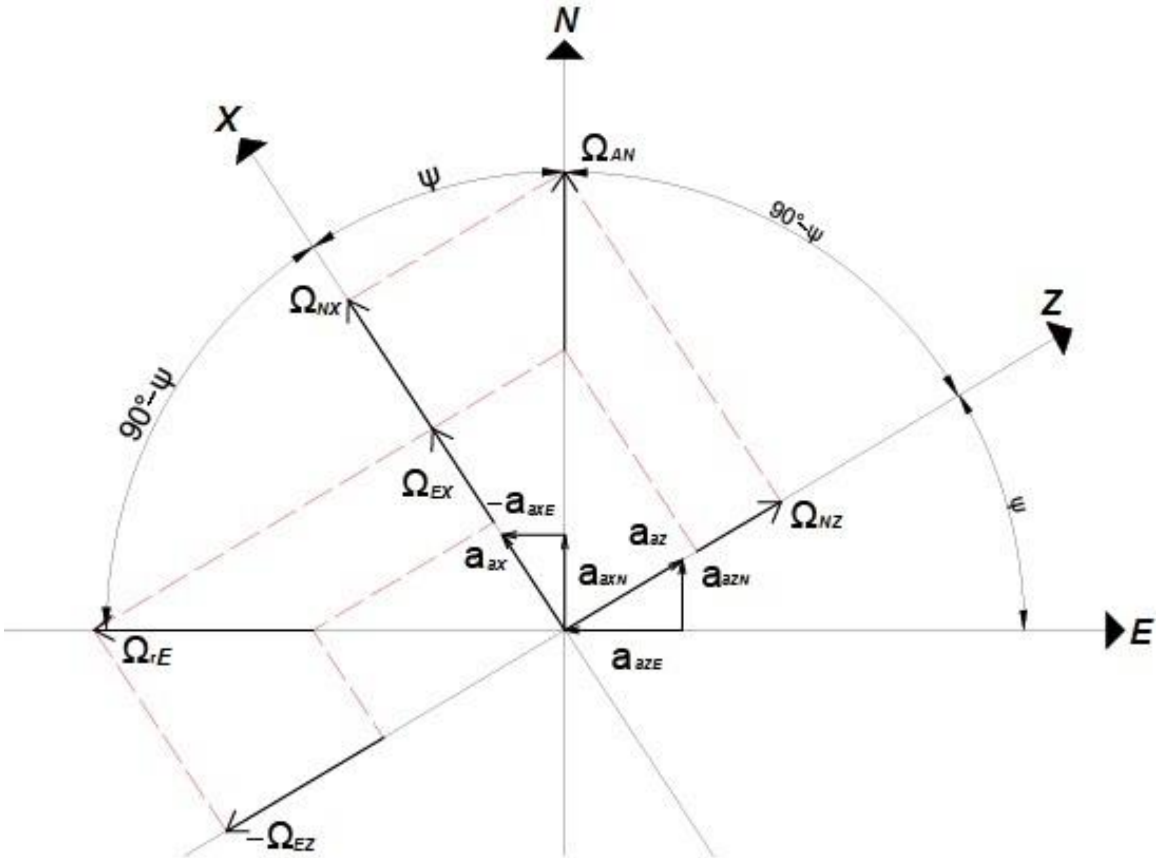

Fig. 3. Transfer angle between the geographical reference system and body-fixed reference system.

Then the transfer of accelerations (21), (22) is as follows:

$$a_{aN} = a_{ax} cos\psi + a_{az} sin\psi; \qquad (21)$$

$$a_{aE} = -a_{ax} sin\psi + a_{az} cos\psi, \qquad (22)$$

where a_{ax} and a_{az} are accelerometer signals along axis *X* and *Z* respectively, and ψ is an angle between the systems.

The analytical bank-and-climb gyro unit can operate normally, as the aircraft's actual heading ψ is an input signal supplied from other aircraft systems. The signal from the accelerometers is supplied to an angular acceleration computer converter.

The aircraft's flight speed components (23), (24) are computed by integrating the converted components of acceleration a_{aN} and a_{aE}:

$$V_{aE} = \int a_{aE} dt; \qquad (23)$$

$$V_{rN} = \int a_{aN} dt. \qquad (24)$$

Afterwards, the angular velocity components of GRS vertical axis *H* (25), (26) can be calculated in the following way:

$$\Omega_{HE} = \frac{-V_{rN}}{R}; \qquad (25)$$

$$\Omega_{HN} = \frac{V_{aE}}{R}. \qquad (26)$$

The gyro rotor reflects the direction of the Earth's local vertical and is mounted on the platform [14]. The gyro can rotate if the moment sensors MM of the gyro generate control moments M_{cA} and M_{cB}. As the gyro rotates relative to the platform reference system *XYZ*, a recalculation should be performed to make the gyro rotate in inertial space like Earth's vertical axis *H* does. Hence, the angular velocity of the Earth's vertical axis should be transferred (27), (28) from the GRS ($\boldsymbol{\Omega_{HN}}$ and $\boldsymbol{\Omega_{HE}}$) to the body-fixed reference system *XYZ*:

$$\omega_{gx} = \omega_{gA} = \Omega_{HN} cos\psi + \Omega_{HE} cos(90 - \psi); \tag{27}$$

$$\omega_{gz} = \omega_{gB} = \Omega_{HN}\, cos(90 - \psi) - \Omega_{HE} cos\, \psi. \tag{28}$$

To perform this particular operation, a second converter section is incorporated in the angular velocity computer.

The last section in the structure scheme of the angular velocity computer is calculation (29), (30) of gyro control moments (M_{VA}, M_{VB}):

$$M_{VA} = H\omega_{gB}; \tag{29}$$

$$M_{VB} = H\omega_{gA}, \tag{30}$$

where

H kinetic moment of the gyro.

The output signals of angular velocity computer, which are proportional to gyro control moments, are supplied to gyro moment sensors. The gyro moment sensors generate moments M_{VA} and M_{VB} affecting the direction of the gyro rotor. The gyro rotates in such a way that its rotor movement is identical to the movement of the Earth's vertical axis *H*. As the gyro rotates, the rotors of angular sensors AS_A and AS_B move relative to its stators. Signals from angular sensors AS_A and AS_B via amplifiers AMP are afterwards sent to platform stabilizing motors SM. Thus the platform is rotating along with the gyro and tracking the gyro's position. Due to the fact that the gyro's rotor is aligned along the Earth's vertical axis *H* and the platform is kept perpendicular to the gyro's rotor, the platform is situated in the Earth horizon. Therefore, the platform reflects the Earth's direction. Selsyns-sensors SS_X and SS_Z are situated on platform axes *X* and *Z* in such a way that Selsyns rotors are connected with the platform, but the stators are attached to the aircraft. When the aircraft is rotating around *X* axis (roll angle) or around *Z* axis (pitch angle) Selsyns-sensors SS_X and SS_Z move relative to its rotors. Because of that, the Selsyns-sensors generate signals, which are proportional to roll and pitch angles.

The errors of the analytical bank-and-climb gyro unit evolve with time due to the operation of integration. For instance, if the accelerometers measure accelerations with errors (Δa_{aE}, Δa_{aN}), aircraft flight speed components (31), (32) are calculated erroneously:

$$\Delta V_{aE} = \int \Delta a_{aE} dt; \tag{31}$$

$$\Delta V_{rN} = \int \Delta a_{aN} dt. \tag{32}$$

Consequently, the components of the angular velocity (33), (34) of vertical axis *H* are calculated erroneously as well:

$$\Delta\Omega_{HE} = \frac{-\Delta V_{rN}}{R}; \tag{33}$$

$$\Delta\Omega_{HN} = \frac{\Delta V_{aE}}{R}. \tag{34}$$

Assuming that the heading remains constant, gyro angular velocity components (35), (36) are also calculated erroneously:

$$\Delta\omega_{gA} = \Delta\Omega_{HN} cos\psi + \Delta\Omega_{HE} cos(90 - \psi); \tag{35}$$

$$\Delta\omega_{gB} = \Delta\Omega_{HN}\, cos(90 - \psi) - \Delta\Omega_{HE} cos\, \psi. \tag{36}$$

From the previous expressions it is clear that the gyro rotor does not rotate in inertial space as the Earth's vertical does. The errors of the analytical bank-and-climb gyro unit (37), (38) are equal to the integral of the difference of gyro rotor axis *C* and the angular velocity of the Earth's vertical axis *H*.

$$\int \Delta\omega_{gA} dt = \Delta\gamma; \quad (37)$$

$$\int \Delta\omega_{gB} dt = \Delta\upsilon. \quad (38)$$

As a consequence, errors of the analytical bank-and-climb gyro unit appear because of two-stage integration and evolve with time [15].

V. Conclusion

The analytical bank-and-climb gyro unit consists of a gyro, accelerometers and an analytical unit. The gyro reflects the direction of the Earth's local vertical. In order to determine the analytical principle of bank-and-climb gyro unit operation, the gyro analysis is required. In order to design the analytical bank-and-climb gyro unit, the East component of the aircraft's absolute flight speed V_{aE} and the North component of the aircraft's relative flight speed V_{rN} should be found. The errors of the analytical bank-and-climb gyro unit appear because of two-stage integration. The errors of the bank-and-climb gyro unit tend to evolve with time.

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