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On the Dynamic Response Prediction at the Full-Scale Test of Aircraft Component

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Abstract **– Several general effects of boundary conditions and their adequate description at structural dynamic computational simulation constitute the main subject of the discussion. First, an analytical model of elastically supported beam was considered to evaluate the effects of support compliance to the basic dynamic characteristics. Second, a more complex model of a body with elastic support was simulated. Some general properties of structure dynamics were analysed.**

Keywords **– Aircraft component, computational simulation, dynamics test, strength.**

I. Introduction

Regardless of the final applied goal, the damage identification in full-scale components of complex mechanical systems implies a complete or partial determination of their dynamic properties. Frequently, such an adequate solution cannot be obtained by using theoretical analysis only. In these cases, an accurate test provides the necessary information to solve the problem of dynamic identification [1]. There are a number of research works and developments in mechanical, civil and aerospace engineering dedicated to the vibration analysis for different applications. A corresponding overview of this information can be found in [2] – [8].

Usually two types of tests are used in practice: forced and ambient vibration tests. The first of them is coupled with the technique of empirical modal analysis (EMA) which is presented in [9]. The EMA technique is more complete and accurate. It helps to identify the dynamic properties of a system, but requires special equipment to excite vibrations. Despite this, the EMA technique has been used in the determination of the dynamic characteristics of even very large structures [10] – [13]. Another method – operational modal analysis (OMA) uses output only. It is cheaper and faster than EMA and can be easily applied to large structures [14].

The computational simulation can be useful for both the test arrangement and interpretation of their results. The computational simulation can improve the effectiveness of the analysis and solve the key problems of structural health monitoring and dynamic system identification [15] – [18], coupling technique [19] and [20], structural integrity [21] and [22], nonlinear dynamics [23], especially, of nonlinear aero elasticity [24]. In the practice of designing and production of new rotorcraft, the full-scale test of the basic prototype is needed before the first flight. This is done due to the requirement to reduce the risk of losing a high-cost item and to minimize the risk of the test operator. In such a test, the reliability of functioning of all rotorcraft systems should be checked, especially, the control system, as well as the strength of the structure at all stages of the flight. However, under the dynamic loading the type and parameters of testing equipment can significantly affect the level and distribution of stresses and strains, as well as their changes over time. In turn, the planning and preparation of these tests requires accurate designing of the test setup, its control system, a preliminary analysis of the dynamic behaviour of the system “test setup – object”.

In this paper, some general effects of the boundary conditions and their adequate description at the structural dynamic analysis are the main subjects of the study. Firstly, the paper considers a simplified model of the elastically supported beam to evaluate the effects of the support compliance to the basic dynamic characteristics. Secondly, a more complex model of the body elastic attachment is discussed.

II. Analytical Study

A. On the Properties of Solutions of the Structural Dynamics of Elastic Systems

The general solution of the linear dynamic problem of an elastic system is described by a system of ordinary differential equations or partial differential equations. Each such set of equations has an infinite number of solutions, among which there is a unique solution to a specific problem. It is determined by the boundary conditions. In other words, the properties of the external and internal constraints are determined by the external supporting and interaction between the parts of the dynamic system. For example, for the one-dimensional problem, the number of permanent integration coincides with the number of superimposed ties. In the practice of real system analysis, the properties of the boundary conditions are often simplified: absolutely rigid supports, perfectly smooth contact surfaces (frictionless). These are the so-called classical boundary conditions. Obviously, the real systems do not have any classical boundary conditions. In each case, the effects of possible deviations should be evaluated. If necessary, the boundary conditions can be described in more detail to provide a correct result.

This paper analyses the ways of obtaining the estimates of the effect of boundary conditions and some general regularities of this effect.

B. Simple Example: A Cantilever Beam With an Elastic Clamping

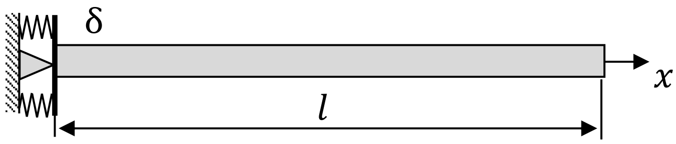

Fig. 1. A cantilever beam with an elastic rotational support.

In this example, the analysis of the system permitting a simple analytic solution is carried out. It allows to show some general regularities of the effect of boundary conditions to the dynamic characteristics of the elastic system.

The transverse free oscillations of the thin uniform beam with elastic support are analysed.

The solution of a differential equation of the beam bending allows to obtain the general solution of the beam shape $V(x)$ of the normal vibration mode

$$V(x) = C_1 \cosh kx + C_2 \sinh kx + C_3 \cos kx + C_4 \sin kx, \tag{1}$$

where k is a root of characteristic equation.

The integration constants C_1, C_2, C_3, C_4 are defined by the boundary conditions. For the cantilever beam (Fig. 1), they can be expressed as follows:

$$V(0) = 0, \quad V'(0) = \delta D V''(0), \;\; V''(l) = 0, \quad V'''(l) = 0.$$

This creates a system of four linear homogeneous algebraic equations for determining the integration constants C_1, C_2, C_3, C_4. This system has a non-trivial solution if the matrix of coefficients is equal to zero. The frequency equation in this case is:

$$\cosh kl \cos kl + \bar{\delta} kl(\cos kl \sinh kl - \sin kl \cosh kl) + 1 = 0, \tag{2}$$

where

$\bar{\delta} = \frac{\delta}{l/D}$	is a relative compliance of the support;
δ	is a rotational compliance of the support;
D	is a bending stiffness of the cantilever beam cross-section.

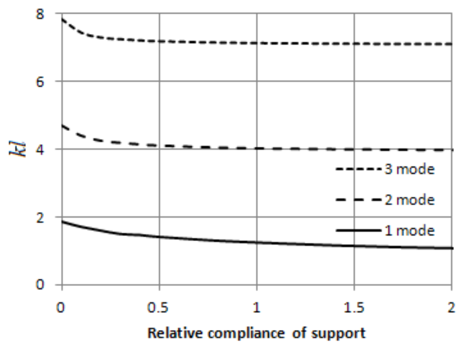

Fig. 2. Natural frequencies as functions of relative compliance of support.

The roots kl of the frequency equation define the spectra of beam eigenfrequencies.

$$f_n = \frac{(kl)_n}{2\pi l^2}\sqrt{\frac{D}{m}}, \tag{3}$$

where

m is a mass of the beam unit length,

$n = 1, 2, \ldots$ is a number of the mode of oscillations.

In Figure 2, the natural frequencies as the function of elastic compliance are presented for the first three modes. A monotonic decrease of all natural frequencies is observed. If the compliance coefficient tends to infinity (disappearance of constraints), the first natural frequency tends to zero, so that the oscillatory form disappears. Higher natural frequencies have nonzero limits, and for higher mode the rate of approaching to this limit is greater. In other words, if the mode of oscillation is higher, the natural frequency of this mode and its shape is less sensitive to a change of the elastic compliance of the support.

C. Dynamic Properties and Response of the Structure

Here is presented a general mathematical description of the complex elastic system that can be released by computational simulation for practical applications. Some elastic body or system of m bodies in the region $W = \bigcup_{j=1}^{m}(W_j)$ bounded by the external surface S are considered. Internal constraints are defined on the subbodies contact surfaces $S_{ij} = S_i \cap S_j$, and the external boundary conditions are given on the other part of surface S. The displacement vector $\boldsymbol{u}(\boldsymbol{x},t)$ is defined by the following motion equation:

$$\rho(\boldsymbol{x})\ddot{\boldsymbol{u}}(\boldsymbol{x},t) = L(\boldsymbol{u}) + \boldsymbol{p}(\boldsymbol{x},t), \tag{4}$$

where

$L(\boldsymbol{u})$ is a linear operator of the displacement vector $\boldsymbol{u}(\boldsymbol{x},t)$;

$\boldsymbol{p}(\boldsymbol{x},t)$ is an intensity of the excitation force;

$\boldsymbol{x}$ is a vector of coordinates of a point.

For example, the operator $L(\boldsymbol{u})$ view of isotropic elastic body is

$$L(\boldsymbol{u}) = \lambda\,\mathrm{grad}(\mathrm{div}\,\boldsymbol{u}) + \mu\Delta\boldsymbol{u}, \tag{5}$$

where

λ and μ are Lame constants.

The equation (3) can be resolved by the separated variables method in the following form:

$$\boldsymbol{u}(\boldsymbol{x},t) = U(\boldsymbol{x})\theta(\mathrm{t}). \tag{6}$$

This solution exists if the function $\boldsymbol{U}(\boldsymbol{x})$ is some eigenmode of the next ordinary differential equation:

$$L(\boldsymbol{U}) + \omega^2\rho(\boldsymbol{x})\boldsymbol{U}(\boldsymbol{x}) = 0. \tag{7}$$

The non-trivial solution $\boldsymbol{U}_k(\boldsymbol{x})$ (shape of the eigenmode) of equation (9) exists for some spectrum of eigenvalues (natural frequencies) ω_k , (k =1, 2, …).

At forced oscillation, the dynamic response of an elastic linear dynamic system under some external load can be described as a modal decomposition of the displacement vector $\boldsymbol{u}(\boldsymbol{x},t)$ to the basic system of functions $\boldsymbol{U}_k(\boldsymbol{x})$ (k = 1, …, ∞). As a result, the vector of displacements can be presented by series

$$\boldsymbol{u}(\boldsymbol{x},t) = \sum_{k=1}^{\infty} \boldsymbol{U}_k(\boldsymbol{x})\theta_k(t), \tag{8}$$

where

$\theta_k(t)$ is a so-called normal function, which is a solution of the following ordinary differential equation:

$$M_k\ddot{\theta}_k(t) + M_k\omega_k^2\theta_k(t) = \Phi_k(t), \tag{9}$$

here

$M_k = \iiint \rho(\boldsymbol{x})\boldsymbol{U}_k^2(\boldsymbol{x})\mathrm{d}V$, $\Phi_k(t) = \iiint \boldsymbol{p}(\boldsymbol{x},t)\boldsymbol{U}_k(\boldsymbol{x})\,\mathrm{d}V$ is a modal mass of the system and a modal force respectively associated with the k-th mode of free oscillations.

The dynamic response $\boldsymbol{u}(\boldsymbol{x},t)$ at a harmonic excitation by the force $\boldsymbol{p}(\boldsymbol{x},t) = \boldsymbol{p}_0(\boldsymbol{x})e^{i\omega t}$ can be expressed by the following series:

$$\boldsymbol{u}(\boldsymbol{x},t) = e^{i\omega t}\sum_{k=1}^{\infty}\frac{\boldsymbol{U}_k(\boldsymbol{x})\Phi_{k0}}{M_k(\omega_k^2-\omega^2)}, \tag{10}$$

where

$\Phi_{k0} = \iiint \boldsymbol{p}_0(\boldsymbol{x})\boldsymbol{U}_k(\boldsymbol{x})\mathrm{d}V$ and $\boldsymbol{p}_0(\boldsymbol{x})$ is an amplitude of modal force.

III. Computational Simulation of the Complex Elastic System

Fig. 3. Non-constrained body of the helicopter.

The application of simulation for the analysis of the dynamic properties of the elastic system (helicopter body) is given here.

Two basic versions of the boundary conditions are compared. The first version is a free body of the helicopter (Fig. 3) that corresponds to the flight. The second one is the same body fixed by a special supporting module (Fig. 4). In this case, this module is connected with the floor of the cargo compartment. The four beams of the module are simulated and fixed by several connection units (Fig. 4, c). The tail part of the helicopter was simulated separately, as the body component with the smallest stiffness. It was assumed that this part was fixed in the contact cross-section with the centrepiece module of the body.

a) b) c)

Fig. 4. Fixed body of the helicopter: a) general view of the helicopter body; b) view of the body attachment to the supporting module in the multiple nodes of power flow; c) contact areas of the basic beams of the fixed attachment.

Some results of full-scale simulation, which was performed by the Autodesk Inventor and its in-build ANSYS are given below for the frequency band upon 20 Hz. The natural frequencies are given in Table I.

TABLE I

RESULTS OF DYNAMIC SIMULATION

Free body	–	–	–	–	–	–	8.28	8.66	9.27	12.01	16.25	16.91	19.99
Fixed body	2.45	3.53	3.96	5.26	6.47	6.9	7.82	8.69	9.8	11.48	14.62	17.42	–
FTB*	–	–	–	–	–	–	7.96	8.00	–	–	–	–	–

*Fixed tail beam.

IV. DISCUSSION

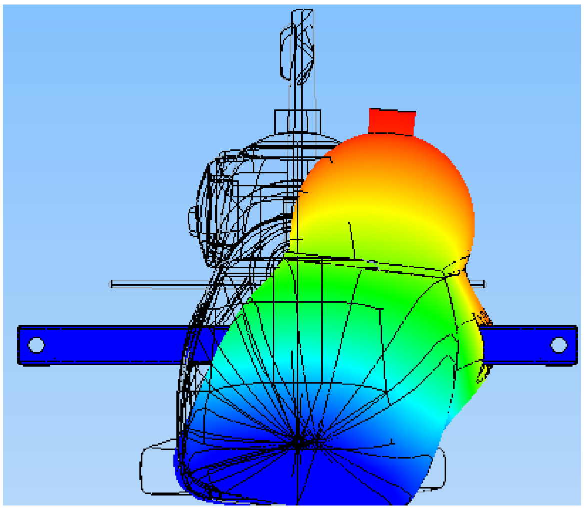

Fig. 5. The shape of the 1st mode of the fixed body (2.45 Hz).

First, the comparison of the natural frequencies spectra of the free and elastically fixed helicopter allows to see the effect of “eigenfrequencies disappearance”. The analysis of this effect is given in [25]. It can be seen from Table 1 that the imposition of elastic constraints on the test object leads to the appearance of six additional eigenfrequencies in comparison with an unattached object. This is explained by the absence of external constraints of an unattached object that causes the conversion of the first six modes of oscillations into six non-oscillatory modes of motion in accordance with the number of degrees of freedom.

It is important to note that the lower modes of the constrained structure are not the pure “rigid body” modes. Figure 5 shows the first mode with a natural frequency of 2.45 Hz. It is seen that this mode combines the lateral translation and rotation with significant deformation of a structure. It follows that the imposition of additional links during the full-scale dynamic test can cause the appearance of dynamic stresses which is not typical for loading in the flight.

It should also be noted that there is no complete identity with the dynamic properties of higher modes, which are not connected with “rigid body” movements. For example, similar vibration mode shapes are for some component of the structure at a test setup and in the flight. For the both cases, corresponding eigenfrequencies can be close. But the significant difference of the shapes is indicated.

Moreover, the isolated rigidly fixed tail part has two natural modes similar to the corresponding modes at the full-scale simulation. However, the total number of higher eigenfrequencies in practically significant frequencies range is less at fixed supporting. It is also obvious that the parametric simulation of the full-scale test structure allows to obtain more adequate dynamic characteristics of the “structure – test setup”.

V. CONCLUSION

First of all, the practical importance of the dynamic structural analysis to optimize the aircraft can be noted. The of the dynamic characteristics of elastic structures is an effective tool for both the structure improvement at designing stage and the rational planning of the full-scale tests of an aircraft or its basic components. The importance of an adequate description of the boundary conditions for the correct outcome is shown. Some basic regularities of the influence of boundary conditions on the dynamic properties of the elastic dynamic system are illustrated. The simple example shows the specific effect of the elastic compliance of constraints. The increase of the elastic compliance of supports reduces the natural frequencies. It can be seen that there is some critical compliance of the support for higher modes. If the compliance is greater than its critical value, the corresponded mode is almost insensitive to the compliance of this constraint. The critical compliance is less for the higher mode. In case of disappearance of some constraints, the lower vibration modes also disappear, and their number is equal to the number of new degrees of freedom. These properties are common to the elastic system of any complexity.

It should be noted that the computational simulation allows to modify the boundary conditions by varying rigidities, masses or damping of the structural system under excitation as well as by producing additional forces. This is especially important for adequate dynamic properties when planning separate component tests of the structural system [26]. In presented article, analysis is focused to comparison different kinds of boundary conditions for the same object. Problem of validation of dynamic simulation is considered in the [26].

References

[1] J. L. Crassidis and John L. Junkins, *Optimal Estimation of Dynamic Systems. Applied Mathematics and Nonlinear Science Series*. Chapman and Hall/CRC, Boca Raton, FL, 2004.
[2] J. P. Conte, X. He, B. Moaveni, S. F. Masri, J. P. Caffrey, M. Wahbeh, F. Tasbihgoo, D. H. Whang, and A. Elgamal, “Dynamic Testing of Alfred Zampa Memorial Bridge,” *Journal of Structural Engineering, ASCE*, vol. 134, issue 6, pp. 871–1066, 2008. https://doi.org/10.1061/(asce)0733-9445(2008)134:6(1006)
[3] B. Peeters, J. Maeck, and G. De Roeck, “Vibration-based damage detection in civil engineering: excitation sources and temperature effects,” *Smart materials and Structures*, vol. 10, pp. 518–527, 2001. https://doi.org/10.1088/0964-1726/10/3/314
[4] P. J. Schubel, R. J. Crossley, E. K. G. Boateng, and J. R. Hutchinson, “Review of structural health and cure monitoring techniques for large wind turbine blades,” *Renewable Energy*, vol. 51, pp. 113–123, 2013. https://doi.org/10.1016/j.renene.2012.08.072
[5] K. He and W. D. Zhu, “Detecting Loosening of Bolted Connections in a Pipeline Using Changes in Natural Frequencies,” *ASME J. Vib. Acoust*., vol. 136, issue 3, pp. 034503, 2014. https://doi.org/10.1115/1.4026973
[6] H. Sohn, C. Farrar, N. Hunter, and K. Worden, “A Review of Structural Health Monitoring Literature: 1996-2001,” Los Alamos National Laboratory report, (LA-13976-MS), 2003.
[7] E. P. Carden and P. Fanning, “Vibration based condition monitoring: A review,” *Structural Health Monitoring*, vol. 3, pp. 355–377, 2004. https://doi.org/10.1177/1475921704047500
[8] C. R. Farrar, S. W. Doebling, and D. A. Nix, “Vibration–based structural damage identification,” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 359, issue 1778, pp. 131–149, 2001.
[9] M. N. Rezai, J. E. Bernard, and J. M. Starkey, “Empirical modal analysis,” *The Shock and Vibration Digest*, vol. 15, 1983.
[10] D. J. Ewins, *Modal Testing: Theory, Practice and Application*. Baldock: Research Studies Press, UK, 2003.
[11] M. W. Halling, I. Muhammad, and K. C. Womack, “Dynamic testing for condition assessment of bridge bents,” *Journal of Structural Engineering, ASCE*, vol. 127, issue 2, pp. 161–167, 2001. https://doi.org/10.1061/(ASCE)0733-9445(2001)127:2(161)
[12] J. M. W. Brownjohn, P. Moyo, P. Omenzetter, and Y. Lu, “Assessment of highway bridge upgrading by dynamic testing and finite- element model updating,” *Journal of Bridge Engineering, ASCE*, vol. 8, issue 3, pp. 162–172, 2003. https://doi.org/10.1061/(ASCE)1084-0702(2003)8:3(162)
[13] M. M. Abdel Wahab and G. De Roeck, “Dynamic testing of prestressed concrete bridges and numerical verification,” *Journal of Bridge Engineering, ASCE*, vol. 3, issue 4, pp. 159–169, 1998. https://doi.org/10.1061/(ASCE)1084-0702(1998)3:4(159)
[14] E. Reynders, “System identification methods for (operational) modal analysis: review and comparison,” *Archives of Computational Methods in Engineering*, vol. 19, pp. 51–124, 2012. https://doi.org/10.1007/s11831-012-9069-x
[15] D. E. Adams, *Health Monitoring of Structural Materials and Components: Methods with Application*. Chichester: John Wiley & Sons Ltd., 2007. https://doi.org/10.1002/9780470511589
[16] V. Giurgiutiu, *Structural Health Monitoring with Piezoelectric Wafer Active Sensors*. Amsterdam & Boston: Elsevier Academic Press, 2008.
[17] G. C. Goodwin and R. L. Payne, *Dynamic System Identification: Experiment Design and Data Analysis*. Academic Press, 1977
[18] S. A. Billings, *Nonlinear System Identification: NARMAX Methods in the Time, Frequency, and Spatio-Temporal Domains*. Wiley, 2013. https://doi.org/10.1002/9781118535561

[19] T. P. Gialamas, D. A. Manolas, and D. T. Tsahalis, “Predicting the Dynamic Behaviour of a Coupled Structure Using Frequency-Response Functions,” *Journal of Aircraft*, vol. 39, issue 1, pp. 109–113, 2002. https://doi.org/10.2514/2.2902
[20] P. Van Loon, Modal Parameters of Mechanical Structures, Ph.D. Dissertation, Dept. of Mechanical Engineering, Katholieke Universiteit Leuven, Leuven, Belgium, 1974.
[21] G. W. Housner, L. A. Bergman, T. K. Caughey, A. G. Chassiakos, R. O. Claus, S. F. Masri, R. E. Skelton, T. T. Soong, B. F. Spencer, J. T. P. Yao, “Structural control: past, present, and future,” *Journal of Engineering Mechanics*, vol. 123, pp. 897–971, 1997. https://doi.org/10.1061/(ASCE)0733-9399(1997)123:9(897)
[22] J. T. Xing, Y. P. Xiong, and W. G. Price, “A generalized mathematical model and analysis for integrated multi-channel vibration structure–control interaction systems,” *Journal of Sound and Vibration*, vol. 320, pp. 584–616, 2009. https://doi.org/10.1016/j.jsv.2008.08.009
[23] G. Dimitriadis, “Bifurcation Analysis of Aircraft with Structural Nonlinearity and Free play Using Numerical Continuation,” *Journal of Aircraft*, vol. 45, issue 3, pp. 893–905, 2008. https://doi.org/10.2514/1.28759
[24] Hua-Peng Chen, “Nonlinear Perturbation Theory for Structural Dynamic Systems,” *AIAA Journal*, vol. 43, issue 11, pp. 2412–2421, 2005. https://doi.org/10.2514/1.15207
[25] Y. G. Panovko and I. I. Gubanova, *Stability and Oscillations of Elastic Systems: Paradoxes, Fallacies, and New Concepts.* Translated by Chas. V. Larrick, New York: Consultants Bureau, 1965.
[26] S. Kuzņecovs, Ē. Ozoliņš, I. Ozoliņš, I. Pavelko, and V. Pavelko, “Dynamic Properties and Fatigue Failure of Aircraft Component,” in *Engineering against Fracture: Proceedings of the 1st Conference*, Greece, Patras, 28–30 May, 2008. Amsterdam: Springer Science, 2009, pp. 105–114. ISBN 9781402094019. e-ISBN 9781402094026. https://doi.org/10.1007/978-1-4020-9402-6_9

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