Hierarchic Optimization of Resource the Resources Allocation among the Units of Aviation Company

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Abstract – **This paper deals with the problem of optimal resource distribution in optimal way among the units of an aviation company. This task could be solved by using the method of dynamic programming. Using Mathcad 14 programming language, there was created a special program that allows to make corresponding calculations. The solution of the real task for an aviation company is observed in this paper as a numerical sample of limited resource distribution between the units of the company in order to get the maximum profit.**

Keywords – **Aviation company, dynamic programming, hierarchic optimization, allocation of resources.**

I. PROBLEM SETTING

One sport aviation company has a small fleet of sport aircraft and its own repair plant with a spare part warehouse. A significant part of the repair parts, which is about 70 % of the total amount, is imported from abroad. At the same time, 30 % of repair parts are produced on the company's own production place and with subcontractors. Besides its own need in spare parts, the company also sells a part of them to other aviation companies and repair shops. In order to minimize storage time, optimize the volume of produced goods and maximize the speed and efficiency of goods deliveries to clients, the company created a supply chain. One of the ideas of this logistic chain is to design together the facility and delivery process to better reveal and support of company own and customer purposes. Using the information technology and efficient flow of information between the members of the chain as well as understanding the business and customer needs, the company provides just in time deliveries of raw materials and ready-made goods [1], [2]. The company's activity would not be successful without good work and cooperation of different departments, which has the following hierarchic structure shown in Fig. 1.

The aviation company consists of two departments: Aircraft department and Repair plant.

The Aircraft department of the company rents a 1 600 square meter warehouse for aircraft storage. Seven employees work in the department: two persons work in the office and five persons work as pilots-instructors. The operational costs of the Aircraft department is about 25 000 EUR per month. The tasks of the Aircraft department are to provide the successful work of the Aviation Sports Club and to support the aircraft rent on a profitable level. Besides, the safety of the flights is a top priority of the company and demands additional support activities from the company's management.

There are 30 employees who ensure the production process of the Repair plant. The company owns a 500 m^2 building and has its own processing equipment. The current production costs are about 30 000 EUR. The main tasks of the Repair plant are to deliver raw materials to the production units, to manage the warehouse of ready-made spare parts (both imported and produced on the spot) and raw materials for processing, to provide the delivery of qualified raw materials just in time and to control the quantity of spare parts in the store without overloading.

The specificity of the production industry lies in its preliminary planning. The production of raw materials has to be planned about six months in advance before the processing. Delays in raw material supply will cause delays in production and ready-made spare part delivery to the repair shop. Besides,

the company sells surplus ready-made spare parts to a number of permanent customers and gets a good profit from these sales. As the warehouse has a limited space for spare part storage, the lean time of the product should be especially as little as possible. The production order is made based on the personal experience of spare part demand and estimations of quantities ordered by customers. After receiving a production order, raw materials and accessories in required quantities have to be bought and delivered to the Repair plant production for further preparation and processing of ready-made products.
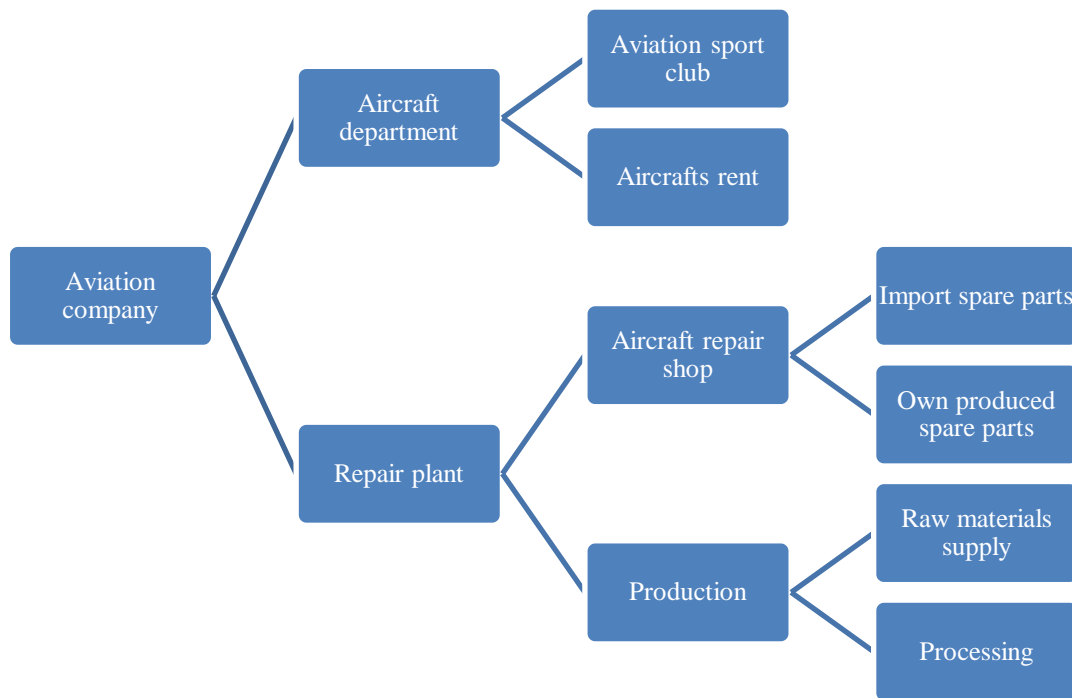

Fig. 1. The Hierarchic Structure of the Aviation Company.

An important task of the aviation company is to combine two processes, each of which has its own specificity and demands: the Aircraft department work and Repair plant with aircraft repair shop and production process.

There is the following sequence of spare part production and aircraft repair.

After evaluating the technical problems of an aircraft and determining the repair steps, the aircraft repair shop using the controlling computer system, Microsoft Dynamics AX, creates an order for the warehouse. The order contains all necessary information about the required spare parts (name, type, model, size, quantity) and delivery terms. Besides the orders from the repair shop, some orders are sent to the warehouse from other companies, which are permanent customers. After getting orders from customers and the repair shop department, the warehouse employees collect and pack goods and form the delivery order for further transportation to the customer and repair shop.

One of the tasks of the warehouse department is to carry out constant monitoring of open (not delivered) sales orders and make an analysis of incoming spare parts (delivery from suppliers) and production volumes. Based on the results of warehouse monitoring, a production order for the required quantity of goods is created for the processing plant, and the estimated time of arrival in stock for each product is calculated. After getting the production order, Processing Manager department evaluates if any raw materials are needed. If there is any lack of raw material stock missing materials should be ordered from the suppliers. After all the accessories for production are at place, the processing of spare parts begins.

Most of the profit for the Sports Aviation Company comes from aircraft rent and sport air flights. In the conditions of economic depression, customer orders is decreasing, which causes aircraft detention and profit losses. To improve the situation, the management of the company has to take some steps to decrease the influence of economic depression on the business and optimize the increasing costs without destroying the working process in the company. To find the right solution of the problem, the management decides to make a profit and loss analysis for various scenarios [3], [4] of the company's development in order to distribute the available financial resources between the departments of the Sports Aviation Company in the most efficient way.

The Sports Aviation Company consists of separate units. We will consider the task of the optimal distribution of limited resources between these units. The company has a hierarchic structure, in other words it means that each unit (besides terminals) has two other units under its control, so in the end we get a treelike structure presented in Fig. 2.
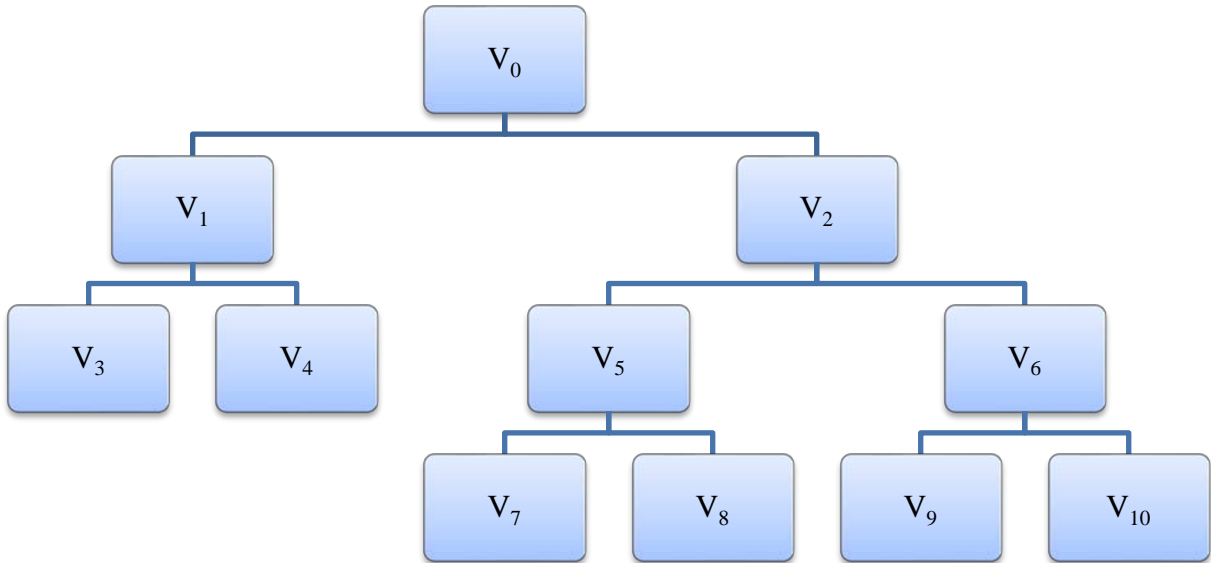

Fig. 2. Treelike Structure.

This structure can be shown by the following relation matrix T (1):

$$T := \begin{pmatrix} 1 & 3 & 5 & -1 & -1 & 7 & 9 & -1 & -1 & -1 & -1 \\ 2 & 4 & 6 & -1 & -1 & 8 & 10 & -1 & -1 & -1 & -1 \end{pmatrix} \tag{1}$$

To make the matrix understandable for Mathcad 14 program, it has to be transposed into the one shown in Table I. The rows of the matrix mean the company's units, while the columns – the number of subordinate units. The column under number "0" shows the number of the unit which is the left son, but the column under number "1" presents the right son. The meaning of the value-1 shows that there is no subordinate unit. So, Table I corresponds to the treelike structure in Fig. 2.

TABLE I

THE RELATION MATRIX T

	0	1
0	1	2
1	3	4
2	5	6
3	–1	–1
4	–1	–1
5	7	8
6	9	10
7	–1	–1
8	–1	–1
9	–1	–1
10	–1	–1

There are eleven numbers of concerned nodes, $n = 11$.

There are various variants of development from 1 to n_i for each i unit. Each j ($j = 1, 2, ..., n_i$) variant assigns the following parameters:

1. concerned the i-th unit's profit $M_{ij}, 0$ from realization of the given variant j. It is significant that the profit depends not only on the resources of the present unit, but also on the resources given to the subordinate units;
2. resource $M_{ij}, 1$ needed for the first subordinate unit (left son);
3. resource $M_{ij}, 2$ for the present unit;
4. resource $M_{ij}, 3$ needed for the second subordinate unit (right son).

Data for each i unit is given by matrix M_i in the same way as it is shown for matrix M_0 bellow in (2). The matrix rows correspond to the offered variants for the present unit. Each row has the above-mentioned parameters for the present variant.

$$M_0 := \begin{pmatrix} 0 & 0 & 0 & 0 \\ 6 & 19 & 3 & 48 \\ 2.5 & 9.1 & 0 & 37 \\ 4.3 & 26 & 2 & 36 \end{pmatrix} \tag{2}$$

The task is to distribute the resources between the units in such a way that the total profit would be maximum.

II. METHOD OF DYNAMIC PROGRAMMING

This task is solved by the method of dynamic programming created by Richard Bellman [5]. The concept of dynamic programming means that a big problem is broken into incremental sub-problems where each sub-problem can be solved and the solution is optimal. As a result, by using a formula, the final solution can be generated without being required to alter the previously solved sub-problems or re-calculating some parts of the algorithm [6]–[8].

The dynamic programming supposes to make a decision step by step. In our case it means to make a decision for each separate unit consequently. Let us accept that the moment of time when a decision should be made is item v, if available resource is equal to r. This resource must be distributed among the item v (father) and all its offspring. Let us have k as various solutions for the unit v.

In this case the Bellman function $F(v,r)$ gives the maximum profit. The value of $F(v,r)$ means the maximum profit that could be derived from the unit v and all its offspring for the resource r.

These calculations should be used starting from the end units and come to the tree root.

Every time Bellman functions are calculated for the units which have sons (subordinate units) with already calculated Bellman function values. The end units have no sons (the condition $T_{v,0} + T_{v,1} = -2$ takes place for the item number v), and the whole resource is given to this unit. Therefore, to apply the j-th project for such a v item, it is necessary to satisfy the condition (3)

$$Mv_{j,2} \leq r. \tag{3}$$

Among the projects for the v-th item, the project $j*$ is chosen for which

$$j* = \arg\{j = 1, ..., n_i: \max\{Mv_{j,0} : Mv_{j,2} \leq r\}\}. \tag{4}$$

For the rest of the units $v = 1, 2, ..., 10$ the condition

$$Mv_{j,1} + Mv_{j,2} + Mv_{j,3} \leq r \tag{5}$$

should be satisfied.

Now among all the projects for the v-th item, the project $j*$ is chosen for which

$$j* = \arg\{j = 1, ..., n_i: \max\{Mv_{j,0} + F(v_0, Mv_{j,1}) + F(v_1, Mv_{j,3}): Mv_{j,1} + Mv_{j,2} + Mv_{j,3} \leq r\}\} \tag{6}$$

where
$v_0 = T_{v,0}$ – is the number of the left son of v;
$v_1 = T_{v,1}$ – is the right son of v.

As a result, the Bellman functions $F(v,r)$ will be calculated for all units and amounts of the given resources $r = 0, 1, …, R$. The above-mentioned procedure is named an inverse algorithm (bottom-up) of dynamic programming.

The direct algorithm (top-down) gives the optimal quantities of assignment resources for each unit v. It is realised in the direction opposite to the above-mentioned – from the root to the end units, each time moving from farther to one of its sons.

The optimal solution for the root (unit with number 0) corresponds to the project $j*$ that satisfies the equality

$$F(0, R) = M_{0j*, 0} + F(1, M_{0j*, 1}) + F(2, M_{0j*, 3}), M_{0j*, 1} + M_{0j*, 2} + M_{0j*, 3} \leq R, \tag{7}$$

because for unit number 0 we have all the resource: $r*(0) = R$

In addition we fix the resource assigned for the sons

$$r(T_{0, 0}) = M_{0j*, 0} , r(T_{0,1}) = M_{0j*, 2} \tag{8}$$

The next step is to go to one of the sons v, knowing the resources given to them. In case when the unit v has the resource r and it has the son $v_0 = T_{v,0}$ (left son) and son $v_1 = T_{v,1}$ (right son), the optimal solution $r*$ for the unit v will be found as a project $j*$ that satisfies the equality

$$F(v,r) = Mv_{j*,0} + F(v_0, Mv_{j*,1}) + F(v_1, Mv_{j*,3}), Mv_{j*,1} + Mv_{j*,2} + Mv_{j*,3} \leq R. \tag{9}$$

The resources assigned for the sons are the following:

$$r(T_{v,0}) = Mv_{j*, 0}, \ \ r(T_{v,1}) = Mv_{j*,2} \tag{10}$$

The direct algorithm is finished when all the points (units) are calculated up to the ones that have no sons.

III. COMPUTER REALIZATION

There is the following information about the computer programs used for the realization of the above-mentioned Bellman algorithm [5], [7].

The programming language we use is a widely known mathematical package Mathcad 14 [9]–[13]. We can create a united matrix M for using it in Mathcad programme:

$$M^T = [M0^T M1^T … M10^T]^T \tag{11}$$

Our next task is to find information concerning a certain unit. To solve it, we create a special Mathcad-program – Locman. Then, for an integer $i,$ Locman gives a number of matrix M row, that is the first row with information about the matrix Mi.

The programme *Father*(v) gives a vector with 2 components, one of which is –1. The second component is an integer one shows the number of unit, which is the father for the unit v. The component location shows which son is the present unit: if it is on the top position, the given unit is the left son; and if it is located bellow, the given unit is the right son.

Let us consider an example:

$$Father(9) = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

Here the father of unit number 9 is unit number 6 and unit number 9 is the left son of unit number 6.

$$Father(10) = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

The father of unit number 10 is unit number 6 and unit number 10 is the right son of unit number 6.

The inverse algorithm [5], [7] of dynamic programming is realized by the program $F(v,r)$, which gives Bellman function value for unit v, which has got the certain resource r. As a result, we get the value of maximum profit and the number of the optimal solution from the matrix M.

The direct algorithm [5], [7] is realized by the program *GlobD*. In other words, *GlobD* gives a matrix of optimal decisions for all the units with a resource equal to *R*. Matrix rows related to the units and contain the optimal value for this unit. Here, r_i is the given for *i* unit, d_i – optimal decision for *i* unit.

IV. NUMERICAL RESULTS

For our example we have the following numeric data mentioned in Table II.

TABLE II

NODES INDICES

$$M1 := \begin{pmatrix} 0 & 11 & 3 & 5 \\ 0.2 & 3.5 & 2 & 4 \\ 1.6 & 16 & 4 & 6 \end{pmatrix} \qquad M2 := \begin{pmatrix} 6 & 14 & 2 & 32 \\ 2.3 & 6 & 0 & 31 \\ 2.7 & 9 & 2 & 24 \end{pmatrix}$$

$$M3 := \begin{pmatrix} 4.6 & 0 & 11 & 0 \\ 4 & 0 & 4 & 0 \\ 7.3 & 0 & 16 & 0 \\ 40 & 0 & 71 & 0 \end{pmatrix} \qquad M4 := \begin{pmatrix} -4.6 & 0 & 5 & 0 \\ -3.8 & 0 & 4 & 0 \\ -5.7 & 0 & 0.57 & 0 \\ 8 & 0 & 90 & 0 \end{pmatrix}$$

$$M5 := \begin{pmatrix} 3 & 6.3 & 0.5 & 7.2 \\ -1 & 3.8 & 1 & 1.28 \\ 1.9 & 4.3 & 1.5 & 3.6 \end{pmatrix} \qquad M6 := \begin{pmatrix} 3 & 18.2 & 1.8 & 12.3 \\ 3.3 & 19.3 & 2.5 & 9.6 \\ 0.8 & 17.6 & 0.56 & 6 \end{pmatrix}$$

$$M7 := \begin{pmatrix} 2.1 & 0 & 6.25 & 0 \\ 0 & 0 & 3.8 & 0 \\ 1.9 & 0 & 4.27 & 0 \end{pmatrix} \qquad M8 := \begin{pmatrix} 0.87 & 0 & 7.25 & 0 \\ -1 & 0 & 1.28 & 0 \\ 0 & 0 & 3.56 & 0 \end{pmatrix}$$

$$M9 := \begin{pmatrix} 1.74 & 0 & 18.25 & 0 \\ 2.32 & 0 & 19.3 & 0 \\ 0.8 & 0 & 17.6 & 0 \end{pmatrix} \qquad M10 := \begin{pmatrix} 1.23 & 0 & 12.3 & 0 \\ 0.98 & 0 & 9.57 & 0 \\ 0 & 0 & 5.6 & 0 \end{pmatrix}$$

The hierarchic structure was presented before in Table I. The information about node indices is presented by Matrix M_0 (see (2)) for 0-unit (root) and matrixes M_1–M_{10} for other units from Table II.

Let us put the global available resource as *R*:= 64. The created program gives the following results: $F(0,R) = 15.9$.

One corresponds to the third solution for item 0 (for the root): $j^* = 3$.

The results obtained for each unit are presented by the matrix *OptD* (see Table III). The matrix columns correspond to the item. The matrix has four rows. The zero row contains the number (*v*) of the items. The first row contains the value of the optimal resource (*r**) for the corresponding item, the second row contains the value of the Bellman function (*F*), the third row contains the number of the optimal solution (*j**) from the matrix *M*.

TABLE III

THE OPTIMAL SOLUTIONS FOR THE RESOURCE OF 64 UNITS (MATRIX *OptD*)

V	0	1	2	3	4	5	6	7	8	9	10
r*	64	26	36	16	6	9	24	0	0	0	0
F(v, r)	15.9	8.9	2.7	7.3	0	0	0	0	0	0	0
j*	3	3	3	3	0	0	0	0	0	0	0

Analysing the results of the calculations presented in the above Table III, we can conclude that the available resource of 64 thousand euro could bring to the company the maximum profit of 15.9 thousand euro in case if the resources will be distributed in the following order (optimal solution number 3 will be taken). So the major part of the resources, 36 thousand euro, should be given to the 2nd unit – the Repair plant, and the smaller part, 26 thousand – to the Aircraft department (the 1st unit), where the Aviation sport club (unit 3) demands most of the support – 16 thousand euros. Own Production (unit 6) usually takes a lot of effort and resources; and in this case, these should be 24 thousand euro. Although the production is a usual cost centre, it will give a profit of 2.7 thousand euro. However, the biggest part of the profit, 8.9 thousand euro, would come from the Aircraft department (unit 1), as this department collects money from customers (sports club, aircraft rental companies and buyers of spare parts) and provides sales for the whole company.

V. CONCLUSION

The problem of resource distribution in an optimal way among the units of the Sports Aviation Company was considered. The Sports Aviation Company has a treelike structure. There are some different ways of developing each unit of the Sports Aviation Company. The ways differ from one another by the amount of necessary resources and profit received. The task has been solved by using the method of *dynamic programming* created by Richard Bellman. During the work, a special program that helps to make calculations was created with the help of Mathcad 14 package.

Using the *dynamic programming* method, the real formulated task for the distribution of necessary resources – 64 thousand euro – among the units of the Sports Aviation Company has been solved and the optimal solution for getting the maximum profit has been found for each unit.

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