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# Analysis of the Strapdown Inertial Navigation System (SINS) Error Genesis

Pjotrs Trifonovs-Bogdanovs[1], Anvar Zabirov[2]

[1, 2] *Institute of Aeronautics, Faculty of Mechanical Engineering, Transport and Aeronautics, Riga Technical University, Riga, Latvia*

***Abstract*** **– Analysis and simulation of the Strapdown Inertial Navigation System (SINS) error genesis revealed that the East Feedback Contour has the greatest influence on the development of an error in this model, and angular velocity sensor $\Delta\omega_y$ is the critical element. In order to prevent the development of an error, structural correction in the East Feedback Contour, and elements that are more critical, namely in angular velocity measurement sensors is the best option.**

***Keywords*** **– Feedback loops, inertial reference system (IRS), SINS error model simulation, SINS simulink analysis, strapdown inertial navigation system (SINS).**

## I. INTRODUCTION

The described Strapdown Inertial Navigation System (SINS) error model with laser gyroscopes was developed in RTU [1]. The system is based on a classic SINS, except the correction for velocity. The main technical characteristics of the experimental data of the system are provided.

Feasibility of new high-precision development SINS is produced by the need to improve the accuracy of object positioning in the context of the possible lack of information through the channels of satellite navigation systems, as well as by the need to autonomously determine the parameters of the angular orientation, angular and linear velocities of an object.

This article describes the SINS error model with velocity correction, and the results of the simulation analysis.

## II. STRAPDOWN INERTIAL NAVIGATION SYSTEM (SINS) ERROR MODEL

Inertial navigation systems are designed to determine the parameters of the spatial position, geographical coordinates and the parameters of aircraft movement data, which is transferred to on-board systems and displays in electronic cabin indicators [2]–[4].

The main structural elements of SINS error model [5]–[8] are (Fig. 1):

- accelerometers;
- angular velocity sensors;
- accelerometers transducer;
- transducer of angular velocity;
- angular velocity calculator;
- angular velocity of angles calculator.

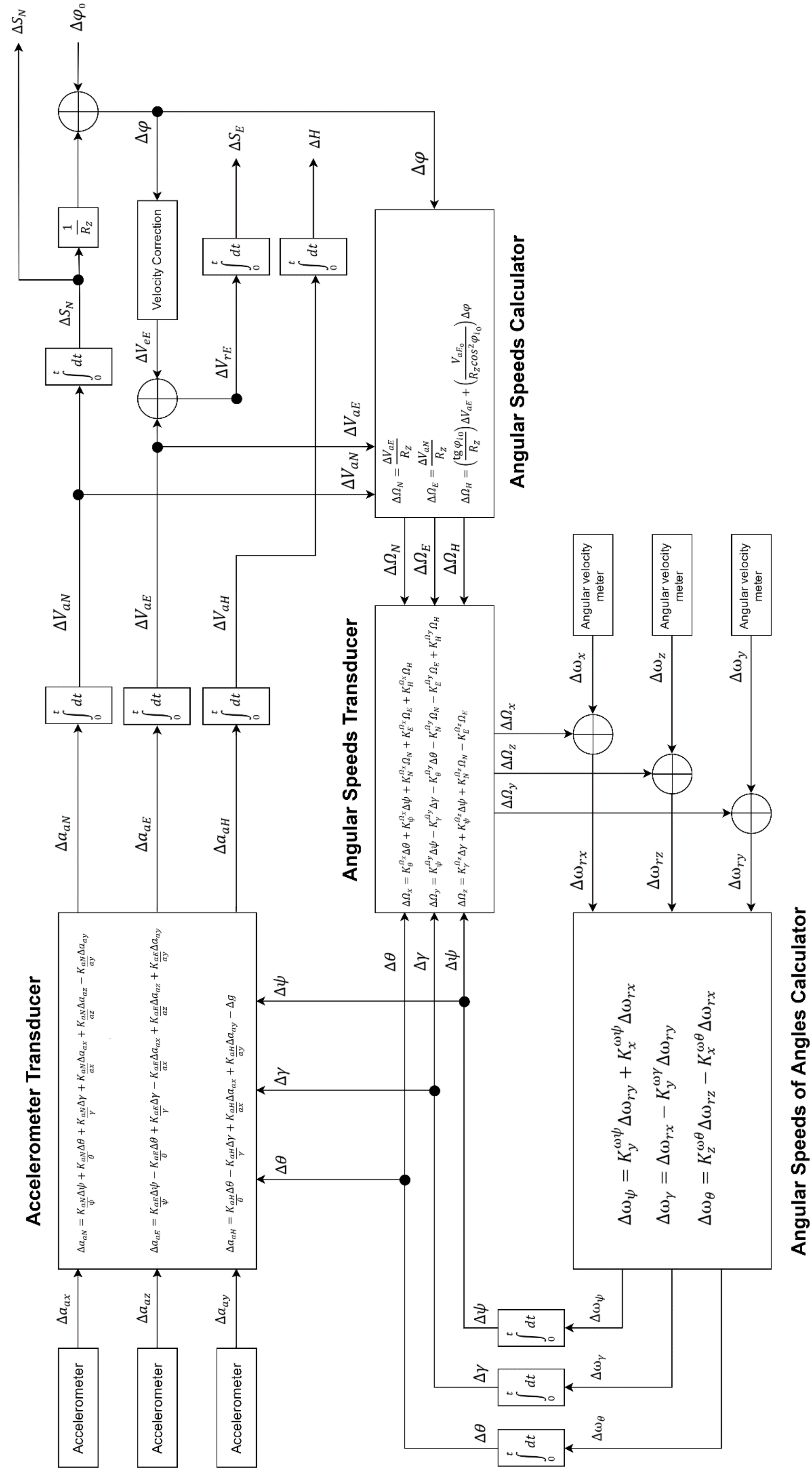


Fig. 1. The Strapdown IRS Error Model with Velocity Correction Block Diagram.

### *A. Accelerometer signals*

Errors in calculating the projections of accelerometer signals – $a_{ax}$, $a_{az}$, $a_{ay}$ on the axis of terrestrial geographical CS – $N$, $E$, $H$ will give the same results as for the original (classical) version SINS (SINS with the coordinates of the output signals of the accelerometers) [9]–[11].

$N$ axis:

$$\Delta a_{aN} = K_{\frac{aN}{\psi}}\Delta\psi + K_{\frac{aN}{\theta}}\Delta\theta + K_{\frac{aN}{\gamma}}\Delta\gamma + K_{\frac{aN}{ax}}\Delta a_{ax} + K_{\frac{aN}{az}}\Delta a_{az} - K_{\frac{aN}{ay}}\Delta a_{ay}; \tag{1}$$

$$K_{\frac{aN}{ax}} = (\cos\psi_0 \cos\theta_0);$$

$$K_{\frac{aN}{az}} = (\sin\psi_0 \cos\gamma_0);$$

$$K_{\frac{aN}{ay}} = (\sin\theta_0 \cos\psi_0 \cos\gamma_0);$$

$$K_{\frac{aN}{\psi}} = \left(\cos\psi_0 \cos\gamma_0\, a_{az_0}\right) - \left(\sin\psi_0 \cos\theta_0\, a_{ax_0}\right) - \left(\sin\theta_0 \sin\psi_0 \cos\gamma_0\, a_{ay_0}\right);$$

$$K_{\frac{aN}{\theta}} = \left(\cos\theta_0 \cos\psi_0 \cos\gamma_0\, a_{ay_0}\right) - \left(\cos\psi_0 \sin\theta_0\, a_{ax_0}\right);$$

$$K_{\frac{aN}{\gamma}} = \left(\sin\theta_0 \cos\psi_0 \cos\gamma_0\, a_{az_0}\right) - \left(\sin\psi_0 \cos\gamma_0\, a_{ay_0}\right).$$

$E$ axis:

$$\Delta a_{aE} = K_{\frac{aE}{\psi}}\Delta\psi - K_{\frac{aE}{\theta}}\Delta\theta + K_{\frac{aE}{\gamma}}\Delta\gamma - K_{\frac{aE}{ax}}\Delta a_{ax} + K_{\frac{aE}{az}}\Delta a_{az} + K_{\frac{aE}{ay}}\Delta a_{ay}; \tag{2}$$

$$K_{\frac{aE}{ax}} = (\sin\psi_0 \cos\theta_0);$$

$$K_{\frac{aE}{az}} = (\cos\psi_0 \cos\gamma_0);$$

$$K_{\frac{aE}{ay}} = (\sin\psi_0 \sin\theta_0 \cos\gamma_0);$$

$$K_{\frac{aE}{\psi}} = \left(\cos\psi_0 \sin\theta_0 \cos\gamma_0\, a_{ay_0}\right) - \left(\sin\psi_0 \cos\gamma_0\, a_{az_0}\right) - \left(\cos\psi_0 \cos\theta_0\, a_{ax_0}\right);$$

$$K_{\frac{aE}{\theta}} = \left(\sin\psi_0 \sin\theta_0\, a_{ax_0}\right) - \left(\sin\psi_0 \cos\theta_0 \cos\gamma_0\, a_{ay_0}\right);$$

$$K_{\frac{aE}{\gamma}} = \left(\cos\psi_0 \cos\gamma_0\, a_{ay_0}\right) - \left(\sin\psi_0 \sin\theta_0 \cos\gamma_0\, a_{az_0}\right).$$

$H$ axis:

$$\Delta a_{aH} = K_{\frac{aH}{\theta}}\Delta\theta - K_{\frac{aH}{\gamma}}\Delta\gamma + K_{\frac{aH}{ax}}\Delta a_{ax} + K_{\frac{aH}{ay}}\Delta a_{ay} - \Delta q; \tag{3}$$

$$K_{\frac{aH}{ax}} = \sin\theta_0\,;$$

$$K_{\frac{aH}{ay}} = \cos\theta_0 \cos\gamma_0\,;$$

$$K_{\frac{aH}{\theta}} = \left(\cos\theta_0\, a_{ax_0}\right) - \left(\sin\theta_0 \cos\gamma_0\, a_{ay_0}\right);$$

$$K_{\frac{aH}{\gamma}} = \cos\theta_0 \cos\gamma_0\, a_{az_0}.$$

*B. Acceleration and Velocity Integration*

$\Delta V_{aN}$ and $\Delta V_{rN}$, $\Delta V_{aE}$ and $\Delta V_{rE}$, $\Delta V_{aH}$ and $\Delta V_{rH}$ errors are the components of absolute and relative velocities in geographic terrestrial CS – $N$, $E$, $H$ [9]–[11].

$$\Delta V_{aN} = \int_0^t a_{aN} dt = \Delta V_{rN}; \tag{4}$$

$$\Delta V_{aE} = \int_0^t a_{aE} dt = \Delta V_{rE} + \Delta V_{eE} = \Delta V_{rE} + (\Omega_z R_z \sin \varphi_{i0}) \Delta\varphi \tag{5}$$

$$\Delta V_{aH} = \int_0^t a_{aH} dt = \Delta V_{rH}; \tag{6}$$

Errors of distance travelled – $\Delta S_N$, $\Delta S_E$, $\Delta S_H$ (in geographic terrestrial CS – $N$, $E$, $H$).

$$\Delta S_N = \int_0^t \Delta V_{rN} dt; \tag{7}$$

$$\Delta S_E = \int_0^t \Delta V_{rE} dt; \tag{8}$$

$$\Delta S_H = \Delta H = \int_0^t \Delta V_{rH} dt; \tag{9}$$

$$\Delta\varphi = \Delta\varphi_0 + \frac{\Delta S_N}{R_Z}. \tag{10}$$

Errors in the components $\Delta\Omega_N$, $\Delta\Omega_E$, $\Delta\Omega_H$ angular velocity of geographic terrestrial CS – $N$, $E$, $H$:

$$\Delta\Omega_N = \frac{\Delta V_{aE}}{R_Z}; \tag{11}$$

$$\Delta\Omega_E = \frac{\Delta V_{aN}}{R_Z}; \tag{12}$$

$$\Delta\Omega_H = \left(\frac{\mathrm{tg}\,\varphi_{i0}}{R_Z}\right) \Delta V_{aE} + \left(\frac{V_{aE_0}}{R_Z \cos^2 \varphi_{i0}}\right) \Delta\varphi_i. \tag{13}$$

$$V_{aE_0} = V_{ABC_0} \sin\varphi_0 + V_Z. \tag{14}$$

Where $V_{ABC_0} = \mathrm{const} = 222\ {}^{m}/_{s^2}$,

$A$ – Lift forces of the aeroplane, $B$ – Air resistance forces of the aeroplane, $C_0$ – Weight/gravity forces of the aeroplane.

$$V_Z = \Omega_Z R_Z. \tag{15}$$

$$V_{aN_0} = V_{ABC_0} \cos\varphi_0. \tag{16}$$

*C. Angular Velocity Transformation*

The algorithm for calculating the projections of composite angles and velocity CS – $N$, $E$, $H$ on axes $x$, $y$, $z$ – in linked CS (Coordinate System) are the same as in classical SINS with the velocity correction, and hence the expressions for $\Delta\Omega_x$, $\Delta\Omega_z$, $\Delta\Omega_y$ will be the same as in the classical version. And in the future, all the options for calculating errors will be the same as in the classical SINS version, since the calculation of angles $\theta$, $\gamma$, $\psi$ at the SINS with the correction for speed is the same as for the classical version of SINS [12]–[15].

For $x$ axis:

$$\begin{gathered}
\Delta\Omega_x = K_\theta^{\Omega x}\Delta\theta + K_\psi^{\Omega x}\Delta\psi + K_N^{\Omega x}\Omega_N + K_E^{\Omega x}\Omega_E + K_H^{\Omega x}\Omega_H; \\
K_N^{\Omega x} = (\cos\theta_0 \cos\psi_0); \\
K_E^{\Omega x} = (\cos\theta_0 \sin\psi_0); \\
K_H^{\Omega x} = \sin\theta_0\,; \\
K_\theta^{\Omega x} = \left(\cos\theta_0\, \Omega_{H_0}\right) - \left(\sin\theta_0 \sin\psi_0\, \Omega_{E_0}\right) - \left(\sin\theta_0 \cos\psi_0\, \Omega_{N_0}\right); \\
K_\psi^{\Omega x} = \left(\cos\theta_0 \cos\psi_0\, \Omega_{E_0}\right) - \left(\cos\theta_0 \sin\psi_0\, \Omega_{N_0}\right).
\end{gathered} \tag{17}$$

For $y$ axis:

$$\begin{gathered}
\Delta\Omega_y = K_\psi^{\Omega y}\Delta\psi - K_\gamma^{\Omega y}\Delta\gamma - K_\theta^{\Omega y}\Delta\theta - K_N^{\Omega y}\Omega_N - K_E^{\Omega y}\Omega_E + K_H^{\Omega y}\Omega_H; \\
K_N^{\Omega y} = \sin\theta_0 \cos\gamma_0 \cos\psi_0\,; \\
K_E^{\Omega y} = \sin\theta_0 \cos\gamma_0 \sin\psi_0\,; \\
K_H^{\Omega y} = \cos\gamma_0 \cos\theta_0\,; \\
K_\psi^{\Omega y} = \left(\sin\theta_0 \cos\gamma_0 \sin\psi_0\, \Omega_{N_0}\right) - \left(\sin\theta_0 \cos\gamma_0 \cos\psi_0\, \Omega_{E_0}\right); \\
K_\gamma^{\Omega y} = \left(\cos\gamma_0 \sin\varphi_0\, \Omega_{N_0}\right) + \left(\cos\gamma_0 \cos\psi_0\, \Omega_{E_0}\right); \\
K_\theta^{\Omega y} = \left(\cos\theta_0 \cos\gamma_0 \sin\psi_0\, \Omega_{E_0}\right) + \left(\cos\theta_0 \cos\gamma_0 \cos\psi_0\, \Omega_{N_0}\right) + \left(\cos\gamma_0 \sin\theta_0\, \Omega_{H_0}\right).
\end{gathered} \tag{18}$$

For $z$ axis:

$$\begin{gathered}
\Delta\Omega_z = K_\gamma^{\Omega z}\Delta\gamma + K_\psi^{\Omega z}\Delta\psi + K_N^{\Omega z}\Omega_N - K_E^{\Omega z}\Omega_E; \\
K_N^{\Omega z} = \cos\gamma_0 \sin\psi_0; \\
K_E^{\Omega z} = \cos\gamma_0 \cos\psi_0\,; \\
K_\gamma^{\Omega z} = \left(\cos\gamma_0 \sin\theta_0 \cos\psi_0\, \Omega_{N_0}\right) - \left(\cos\gamma_0 \cos\theta_0\, \Omega_{H_0}\right) + \left(\cos\gamma_0 \sin\theta_0 \sin\psi_0\, \Omega_{E_0}\right); \\
K_\psi^{\Omega z} = \left(\cos\gamma_0 \cos\psi_0\, \Omega_{N_0}\right) + \left(\cos\gamma_0 \sin\psi_0\, \Omega_{E_0}\right).
\end{gathered} \tag{19}$$

*D. Angular Velocity Transformation and Its Integration*

For the errors in components $\omega_{rx}$, $\omega_{rz}$, $\omega_{ry}$ relative to angular velocities – $x$, $z$, $y$ – $\omega_x$, $\omega_z$, $\omega_y$:

$$\omega_{rx} = \Delta\omega_x - \Delta\Omega_x; \tag{20}$$

$$\omega_{rz} = \Delta\omega_z - \Delta\Omega_z; \tag{21}$$

$$\omega_{ry} = \Delta\omega_y - \Delta\Omega_y. \tag{22}$$

Angular velocity errors $\Delta\omega_\theta$, $\Delta\omega_\gamma$, $\Delta\omega_\psi$ and the change of angles $\theta$, $\gamma$, $\psi$.
Pitch – $\theta$ (Angular velocity):

$$\begin{gathered} \Delta\omega_\theta = K_z^{\omega\theta}\Delta\omega_{rz} - K_x^{\omega\theta}\Delta\omega_{rx}; \\ K_z^{\omega\theta} = \cos\gamma_0\,; \\ K_x^{\omega\theta} = \cos\theta_0 \sin\psi_0. \end{gathered} \tag{23}$$

Roll – $\gamma$ (Angular velocity):

$$\begin{gathered} \Delta\omega_\gamma = \Delta\omega_{rx} - K_y^{\omega\gamma}\Delta\omega_{ry}; \\ K_y^{\omega\gamma} = \cos\gamma_0 \sin\theta_0. \end{gathered} \tag{24}$$

Yaw – $\psi$ (Angular velocity):

$$\begin{gathered} \Delta\omega_\psi = K_y^{\omega\psi}\Delta\omega_{ry} + K_x^{\omega\psi}\Delta\omega_{rx}; \\ K_y^{\omega\psi} = \cos\gamma_0 \cos\theta_0\,; \\ K_x^{\omega\psi} = \sin\theta_0. \end{gathered} \tag{25}$$

Angle error $\theta$, $\gamma$, $\psi$ – $\Delta\theta$, $\Delta\gamma$, $\Delta\psi$:

$$\Delta\theta = \int_0^t \Delta\omega_\theta dt; \tag{26}$$

$$\Delta\gamma = \int_0^t \Delta\omega_\gamma dt; \tag{27}$$

$$\Delta\psi = \int_0^t \Delta\omega_\psi dt. \tag{28}$$

## III. Simulation and Results

In SINS error model with correction on velocity, there are both positive and negative feedback loops, which include two series-connected integrators. The presence of positive feedback leads to the development of an error in the system, and negative feedback produces stabilization of the system (Fig. 2). The main objective is to analyse the SINS error model and to determine the most critical feedback loop with the largest contribution of an error to this system.

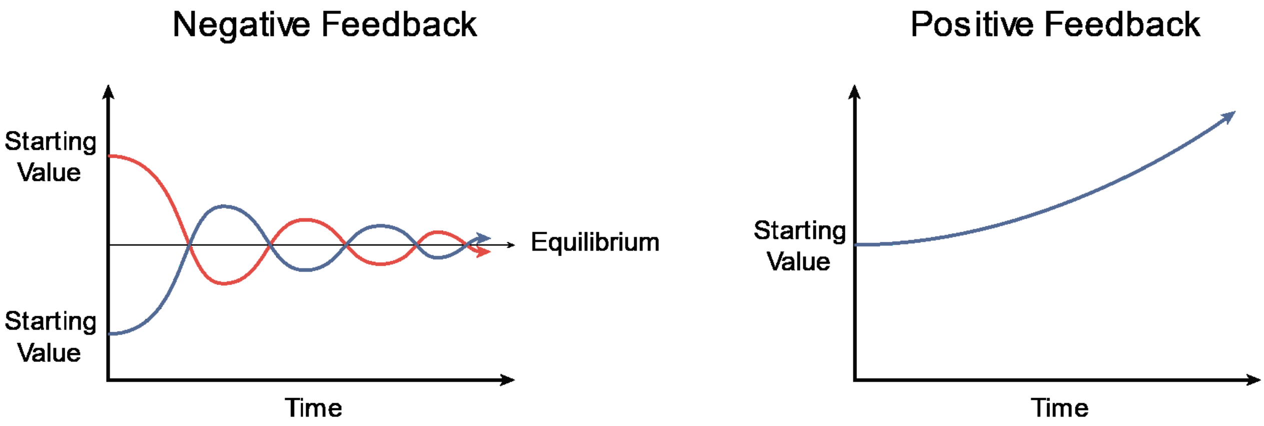


Fig. 2. Negative and Positive Feedbacks.

To determine the feedback loops, which are the most critical, the SINS error model with velocity correction was implemented in Simulink program (Fig. 3).

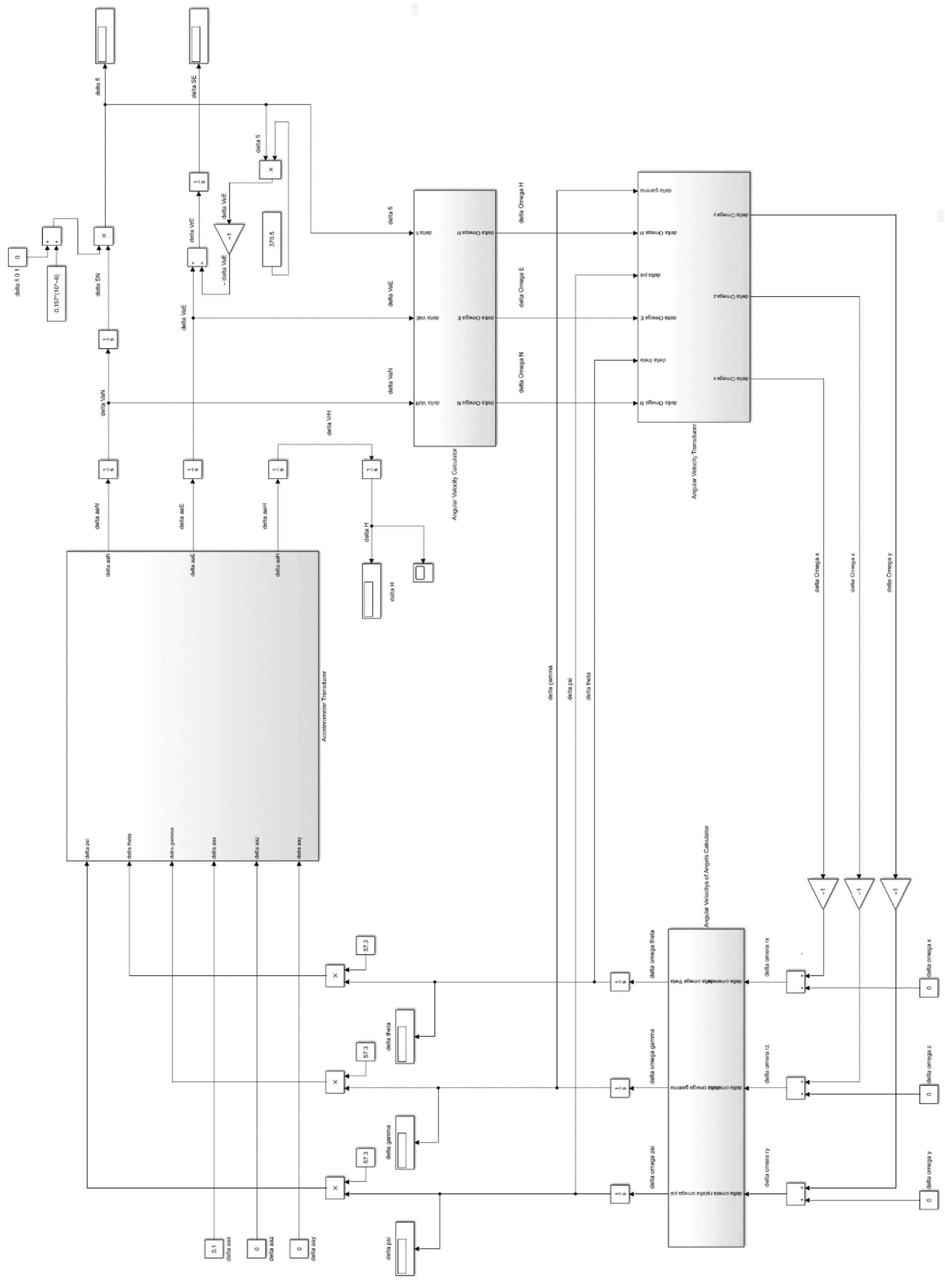

Fig. 3. Simulink Model of the Strapdown IRS Error Model with Velocity Correction.

SINS error model includes 3 channels: North Feedback Channel (NFC), East Feedback Channel (EFC) and North Feedback Channel 2 (NFC2). NFC consists of 6 feedback loops, EFC – 16 feedback loops and NFC2 – 5 feedback loops. During the simulation were launched error to accelerometers and angular velocity sensors, equal to 1 m/s$^2$ and 1°, respectively. 50 different simulation scenarios were considered and processed to determine the most critical feedback loop.

The obtained data were recorded, analysed, and compared with each other, and with the results obtained during the simulation of the entire system (Fig. 4).

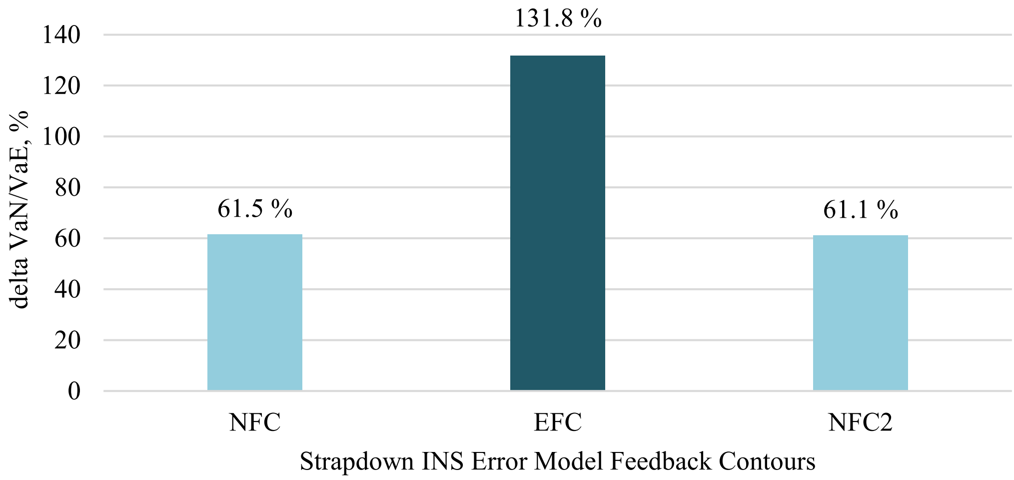


Fig. 4. Influence of all Feedback Contour on SINS Error Model.

Thus, based on simulations it was possible to determine that the most critical feedback loop, which had the greatest impact on the error development in SINS error model with correction on velocity which is EFC10 (Fig. 5).

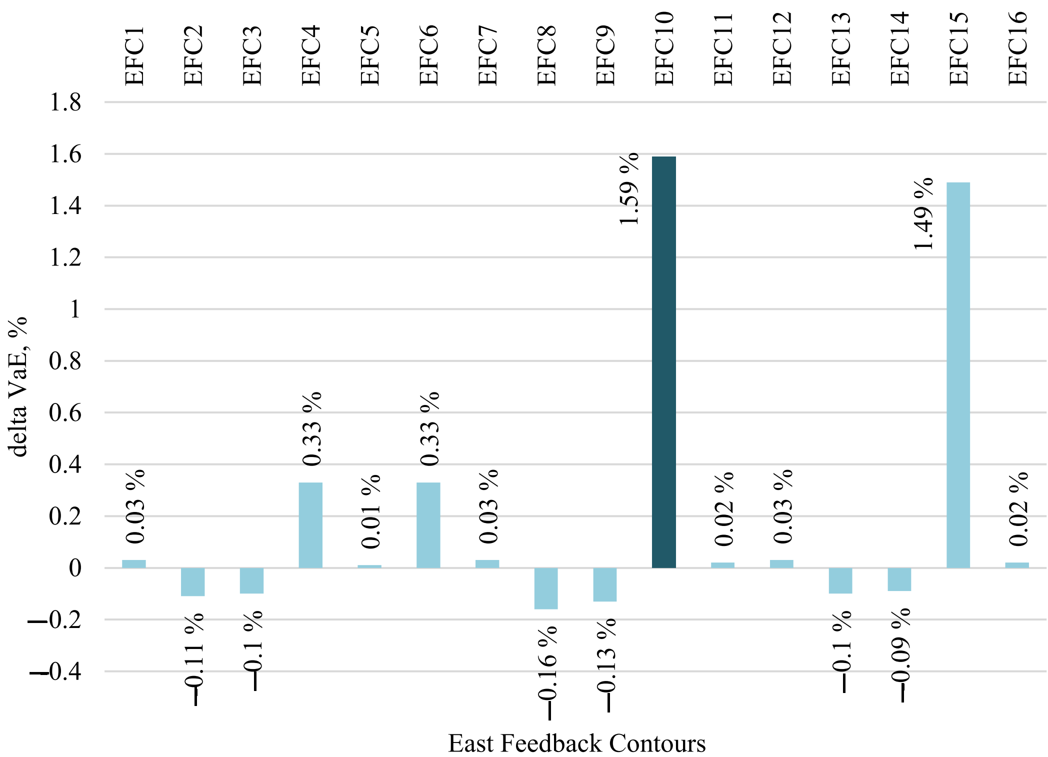


Fig. 5. Influence of all separate East Feedback Contours through Angular Velocity Sensor error on Full EFC (Total).

## IV. THE MODERN SINS SPECIFICATIONS

Table I presents Accuracy Characteristics of the modern SINS-T. SINS-T was developed in AS "MIEA" and is a part of onboard equipment of modern military and civil aircrafts.

TABLE I

SINS-T SPECIFICATIONS

Fig. 6. SINS-T.

| | |
|---|---|
| Name | SINS-T |
| Mass, kg | 15 |
| Dimensions, mm | 197×227.3×410.5 |
| Accuracy Characteristics | |
| In offline mode | 3.7 km per flight hour |
| In correction mode (GPS) | < 30 m |
| Time between failures, hours | 10 000 |

Based on the data and values obtained from simulation we can make conclusion related on the influence of error development of exact feedback loop on SINS. The influence of East Feedback Loop is 131.8 %, which is 4.9 km per flight hour. Due to the presence of negative feedback loops this value is reduced to approximately 60 % (2.9 km per flight hour) [1-8]. The influence of East Feedback Loop on SINS-T model is 54 % (2 km per flight hour) in offline mode and 16.2 m per flight hour in correction mode.

## V. CONCLUSION

In the conclusion of the conducted research on the analysis of error genesis in the simulation and comparison of the feedback loops of the SINS error model with velocity correction by entering an error in the accelerometers and angular velocity sensors it was revealed that the eastern feedback loop exerts the greatest influence on the development of error in this model since it contains the greatest number of positive feedbacks. After analysing the results, was defined that the greatest influence on the development of error is EFC10 loop, and the critical element is angular velocity sensor $\Delta\omega_y$. The analysis shows that in order to prevent the development of an error, structural correction is best served both in the chain and in the elements that are more critical.

## REFERENCES

[1] P. Trifonovs-Bogdanovs, *Basics of Inertial Navigation Basics*. Riga, Latvia: RKIIGA, 1984.
[2] D. Titterton and J. Weston, *Strapdown Inertial Navigation Technology*, 2nd ed. United Kingdom: The Institution of Electrical Engineers, 2005, pp. 1–55, 153–186, 263–274. doi: https://doi.org/10.1109/MAES.2005.1499250
[3] M. S. Grewal, A. P. Andrews, and C. G. Bartone, "Fundamentals of Inertial Navigation", in *Global Navigation Satellite Systems, Inertial Navigation, and Integration*, 3rd ed. New York, USA: Wiley, 2013, pp. 54–103.
[4] P. D. Groves, "Introduction", "Kalmas Filter-Based Estimation", GNSS: User Equipment Processing and Errors", in *Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems*. Artech House, 2013. pp. 1–20, 81–35, 349–386.
[5] K. R. Britting, *Inertial Navigation Systems Analysis*, 3rd ed. New York, USA: Wiley, 2002, pp. 5–67, 72–120.
[6] P. V. Bromberg, *Theory of Inertial Navigation Systems*. Moscow: Nauka, 1979.
[7] *SSJ-100 Maintenance Manuals*. Sukhoi, Alenia Aermacchi, 2016, Chapter 34.
[8] A. P. Sage, *Optimum Systems Control*, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 2001, pp. 3–34, 45–51, 78–101.
[9] V. V. Matveev, *Osnovy postroenija besplatformennyh inercial'nyh sistem*. Saint-Petersburg: Jelektropribor, 2009, pp. 15–77.
[10] V. Branec and I. Shmyglevskij, *Vvedenie v teoriju besplatformennyh inercial'nyh navigacionnyh sistem*. Moscow: Nauka, 1992, pp. 56–67.

[11] G. Fridlender, *Inercial'nye sistemy navigacii*. Moscow: Gosudarstvennoe izdatel'stvo fiziko-matematicheskoj literatury, 1961, pp. 17–29.

[12] R. M. Rogers, *Applied Mathematics in Integrated Navigation Systems*, 3rd ed. AIAA, 2007, pp. 7–59, 73–101. https://doi.org/10.2514/4.861598

[13] E. B. Magrab, B. Balachandran, and S. Azarm, *An Engineers Guide to MATLAB*. United Kingdom: Pearson Education, 2011, pp. 5–23.

[14] P. I. Kattan, *MATLAB for Beginners: A Gentle Approach*. Petra books, 2013, pp. 9–21, 135–153, 153–167.

[15] A. A. Sirota, *Metody i algoritmy analiza dannyh i ih modelirovanie v MATLAB*. BHV-Peterburg, 2016, pp. 7–115, 217–285.

**Pjotrs Trifonovs-Bogdanovs**, Prof. *Dr. habil*. (engineering).
Date and place of birth: 1943, Riga, Latvia.
Education: 1966, Riga Civil Aviation University and Latvian University; Latvian State University, Physical (1979).
Affiliations and functions: 1999 – Habil. Doctor of Engineering Sciences, Aviation Institute of Riga; 2008 – professor, Aviation Institute of Riga Technical University.
Research interests: engineering psychology, systems of automatic controlling, inertial navigation
Publications: 46 science and methodological publications.
Present position: professor at Aviation Institute of Riga Technical University.
Address: Institute of Aeronautics, Faculty of Mechanical Engineering, Transport and Aeronautics, Riga Technical University, Lomonosova 1A, k-1, Riga, LV-1019, Latvia.
Phone: +371 2606 5007
E-mail: ptb@inbox.lv

**Anvar Zabirov** (b. 1991) graduated from Riga Technical University, “Aviation Transport”, in 2016. He received a Master’s degree in 2018 and now he is a Ph. D. student of “Transport” Department of Riga Technical University.
Since 2017 he has worked in RAF-AVIA aircraft maintenance and repair organization as an Avionic engineer.
Address: Institute of Aeronautics, Faculty of Mechanical Engineering, Transport and Aeronautics, Riga Technical University, Lomonosova 1A, k-1, Riga, LV-1019, Latvia.
Phone number: +371 22113199
E-mail: anvar.zabirov@edu.rtu.lv