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The Dependence of the Optimal Size of a Wind Turbine Tower on Wind Profile in Height

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Abstract **– In this article the necessity of measuring wind at its different heights is discussed. The economical optimization of tower height is impossible without such measurement. In places where land is relatively flat, for example, in deserts or swamps, smaller wind turbines are more profitable, while in forest zones bigger turbines are more profitable. In both cases, to make a correct decision on the optimal tower height, it is very important to know exactly the wind profile law. Even for places where land is expensive, the measurement of the wind at different heights can influence the correct decision regarding the optimal size and number of needed turbines for getting the required power. For places with cheaper land, this dependence is even stronger. This analysis refutes a common misconception: "the bigger – the better".**

Keywords **– payback period, return on investment, wind energy conversion, wind profile power law.**

I. Introduction

There are very few alternative sources of energy in Latvia: there are no mountain rivers, oil or coal; dams on flat rivers flood large areas; the extraction of peat destroys the soil; biofuels are labour intensive. After Chernobyl and Fukushima nuclear accidents, it is better to switch from use of nuclear power. Sunny days suitable for solar energy in Latvia last only two months per year: July and August. But in winter, when the need for energy is higher, there is almost no sun at all. Here another source of energy should be considered such as wind. Use of wind energy in Latvia possibly should be the priority. Prior the installation several parameters of wind turbines must be considered and optimized such main parameters as tower height, rated power.

Obviously, the first major optimization factor is the size of the wind turbine (See Fig.1).

Fig. 1. Comparison of two scale options of wind turbines.

II. WIND CHANGE DEPENDING ON HEIGHT

The question of the dependence of wind on height is not well discussed in literature. This is a complex issue: in mathematics, there is a simple power law [1], which is a relationship between the wind speeds U_{10} at 10 meters height, and the speed U_Z at another height Z.

$$U_Z / U_{10} = (Z/10)^K, \quad (1)$$

where K – power law factor, or power law coefficient, or scaling exponent.

The dependence of wind on height is determined by ground obstacles such as: bushes, trees, hills and buildings. That is why the power coefficient can be different, for example: about 0.5 over a forest, about 0.2 over a swamp, and about 0.05 over the sea.

According to [2] it is possible to assume that for the majority of land locations

$$0.14 < K < 0.5 \quad (2)$$

This range is big, and it means that K should be measured locally before the construction starts.

Therefore, such an experimental study should be conducted in Latvia, and the power law factor K should be determined precisely at sites of the possible construction of wind generators. Obviously, the results will vary in various places: swamps, seashores or forests will give different K.

III. HOW THE COST OF WIND POWER TURBINES DEPENDS ON ITS LINEAR SIZE

A. Market prices

In open literature it is difficult to find the direct dependence between the size and price of a wind generator. Only the benefits are advertised [3]. Relying on [4], small systems can also be profitable. Recommendations for the installation of medium size towers for wind turbines are given by Ian Woofenden [5]. His method is much cheaper than commonly used in Germany for big towers, meanwhile the tower costs two to ten times as much as the turbine, depending on the site and condition. The data about wind turbine prices is also contradictive: [6] says: "Wind turbines have significant economies of scale. The costs for a utility scale wind turbine range from about $1.3 million to $2.2 million per MW of nameplate capacity installed. Most of the commercial-scale turbines today are have power 2 MW and cost roughly $3–$4 million installed. Wind turbines under 100 kW cost roughly $3 000 to $8 000 per kW of capacity. A 10-kW machine (the size needed to power a large home) might have an installed cost of $50 000–$80 000 (or more)". On the other hand, it is easy to find 3 kW wind turbines for only $3 000 [7]. Only $1 000 per kW of capacity, i.e. $1 million per MW. Thus, smaller turbines appear to be more profitable.

B. Prices change with time

According to Blumberg New Energy France, the average turbine price by delivery date was $2 000/kW in 2009 but dropped sufficiently to $1 000/kW in 2017 [8]. According to the report [9], wind turbine equipment prices in 2017 fell even to $750/kW. But it is difficult to evaluate if it happens due to real improvements in technology or due to state subsidies.

C. Theoretical analysis of prices

Wind turbines can have significant economies of scale, but only up to a certain size. In the other words: there is always an economically optimal size.

It is possible to scale the wind power generator, making it bigger or smaller. The cost of a single wind power turbine will be some function of its linear size L. (It does not matter if the L is the height of the tower or the diameter of the propeller).

The Taylor set for this function is used:

$$\text{Cost}(L) = A_0 + A_1 \cdot L + A_2 \cdot L^2 + A_3 \cdot L^3 + A_4 \cdot L^4 + \ldots \quad (3)$$

where,

A_0 – costs of anemometer and other control devises which are the same for big and small systems;

$A_1 \cdot L \approx 0$ – cost of electrical cables, hawsers, and other cheap linear parts;

$A_2 \cdot L^2$ – cost of land and surface painting costs;

$A_3 \cdot L^3$ – cost of construction materials, which are proportional to the volume;

$A_4 \cdot L^4$ – crane equipment operating cost. (The load proportional to L^3 must be raised to a height L, for the total job to be done, so the crane job will be proportional to $L^3 \times L = L^4$). Insurance and the cost of any future maintenance or repair can grow like L^4.

IV. Dependence of Electric Power on the Linear Size *L* of Wind Turbine

According to [10], the performance of a turbine will be determined by the wind regime and by its power curve (see Fig. 2).

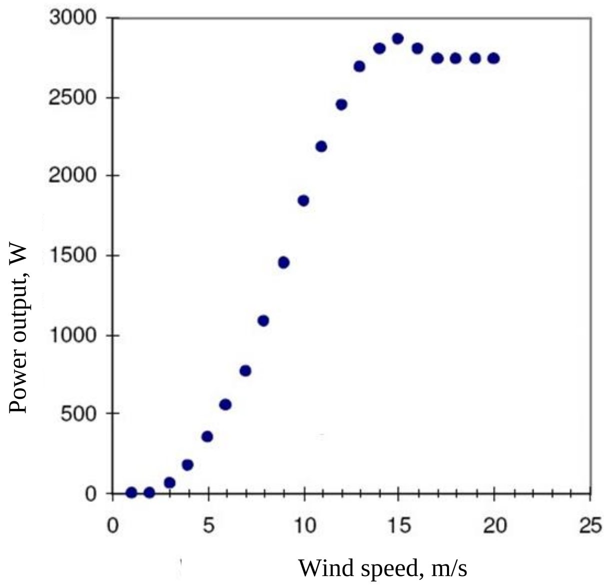

Fig. 2. A turbine's performance will be determined by its power curve [10].

A slight drop at the end occurs due to the transition of the turbine into protective mode. Taking into consideration that for most time only the beginning of this curve is used, some simplification can be accepted: it is possible to assume that produced electric power is proportional to the cube of wind speed U^3 and to the cross-sectional area of the wind turbine, which is proportional to L^2. So, using formula (1) and assuming $Z = L$, the generated power is:

$$\text{P} = N \cdot L^{(2+3K)}, \quad (4)$$

where N – coefficient of proportionality.

Taking into consideration the kilowatt-year price of electricity M, the annual income from selling electricity is

$$\text{Income} = M \cdot N \cdot L^{(2+3K)}. \quad (5)$$

At this step, the possible economic Return on Investment (ROI) can be calculated:

$$\text{ROI} = \text{Income}(L) \,/\, \text{Cost}(L) \sim L^{(2+3K)} \,/\, \text{Cost}(L). \quad (6)$$

To make calculations easier PP – Payback Period can be used instead.
Which is:

$$PP = 1/ROI \sim Cost(L)/L^{(2+3K)}. \qquad (7)$$

Taking into consideration (3):

$$PP \sim A_0\, L^{(-2-3K)} + A_1 \cdot L^{(-1-3K)} + A_2 \cdot L^{(-3K)} + A_3 \cdot L^{(1-3K)} + A_4 \cdot L^{(2-3K)}. \qquad (8)$$

The following calculations for big and small wind turbine constructions differ.

1. Calculations for big wind turbine constructions

In case of big wind turbine constructions, the first two values are negligible:

$$PP \sim A_2 \cdot L^{(-3K)} + A_3 \cdot L^{(1-3K)} + A_4 \cdot L^{(2-3K)}. \qquad (9)$$

For very big wind power generators, when only crane equipment operating cost dominates, the Playback Period is:

$$PP \sim L^{(2-3K)}. \qquad (10)$$

Often in practice $K = 0.333$, so PP increases linearly as L, in other words ROI falls as L^{-1}.
The tendency is obvious: after certain turbine's size the crane equipment operating cost (A_4) will play major role, and big wind power generator becomes less profitable than a small one.
Logically, there exists some optimal size (L) which brings the biggest ROI or, in other words, the shortest PP.
From (3) the optimal size of wind power generator can be calculated by determining the minimum of the function. The derivative is equated to zero:

$$d\,(PP)\,/dL = 0. \qquad (11)$$

Differentiating the (11), the following result is obtained:

$$(-3K) \cdot A_2 \cdot L^{(-1-3K)} + (1-3K) \cdot A_3 \cdot L^{(-3K)} + (2-3K) \cdot A_4 \cdot L^{(1-3K)} = 0\,. \qquad (12)$$

Multiplying by $L^{(1+3K)}$, it leads to a quadratic equation:

$$(-3K) \cdot A_2 + (1-3K) \cdot A_3 \cdot L + (2-3K) \cdot A_4 \cdot L^2 = 0 \qquad (13)$$

or

$$L^2 - \left(\frac{A_3}{A_4}\right)\left(\frac{1}{2-3K} - 1\right)L + \left(\frac{A_2}{A_4}\right)\left(\frac{2}{2-3K} - 1\right) = 0\ . \qquad (14)$$

Solving this equation, it is possible to obtain the economically optimal size:

$$L = \left(\frac{A_3}{2\,A_4}\right)\left(\frac{1}{2-3K}-1\right) + \sqrt{\left(\frac{A_3}{2\,A_4}\right)^2\left(\frac{1}{2-3K}-1\right)^2 - \left(\frac{A_2}{A_4}\right)\left(\frac{2}{2-3K}-1\right)}. \qquad (15)$$

Traditionally, in the solution of the quadratic equation, the sign ± appears before the root. But here, if $0 < K < 0.666$, the negative sign before the root would lead to negative L, which makes no sense. It is known, that in all reasonable locations $0 < K < 0.5$. That is why only a positive value is considered.

Conclusion: The optimal size of a wind generator strongly depends on the wind profile power law coefficient K. This means that the coefficient K for a specific location should be measured first.

Economical calculations the formula requires simplification. The A_2 mostly depends on the price of land. There are two extreme subcases:

1.1. In locations where land is cheap $A_2 = 0$ and $K > 0.333$, like forests, the optimal size is:

$$L = \left(\frac{A_3}{A_4}\right)\left(\frac{3K-1}{2-3K}\right). \qquad (16)$$

This value is positive if $0.333 < K < 0.666$. The price of materials ($A_3 \cdot L^3$) competes with the crane operating cost ($A_4 \cdot L^4$). And the wind distribution law factor K strongly influences the result.

Taking in consideration possible values $0.14 < K < 0.5$ (advised in [10]), the need of measurement of local K becomes clear. In any cases $0 < L < (A_3/A_4)$.

1.2. In locations where land is very expensive, the term with A_2 dominates in equation (16):

$$L = \sqrt{\left(\frac{A_2}{A_4}\right)\left(\frac{3\,K}{2-3\,K}\right)}. \qquad (17)$$

If the price of land $A_2 \cdot L^2$ competes with the crane operating cost $A_4 \cdot L^4$ than the wind distribution power factor K has strong influence on the result. If $0.14 < K < 0.5$, it gives following range for the optimal L:

$$0.52\sqrt{\frac{A_2}{A_4}} < L < 1.73\sqrt{\frac{A_2}{A_4}}. \qquad (18)$$

This means that K values three times differ.

Conclusion: To make a correct decision about the optimal height of wind turbines, the average distribution of the wind at the heights (for which the K factor is responsible) must be taken into consideration.

2. Calculation for small wind turbine constructions

In locations where land is cheap $A_2 = 0$ and $K << 0.333$, like swamps, the turbine can be so small that the crane operating cost $A_4 = 0$.

Such small turbines can be effective only if K is small. Such conditions exist in deserts and swamps, where $K < 0.333$, according to (3), L is:

$$\mathrm{Cost}(L) = A_0 + A_3 \cdot L^3, \qquad (19)$$

$$\mathrm{PP}(L) = \mathrm{Cost}(L) \,/\, \mathrm{Income}(L) = A_0 \cdot L^{(-2-3K)} + A_3 \cdot L^{(1-3K)}. \tag{20}$$

The optimal size of wind power generator can be calculated by determining the minimum of function (11):

$$(-2-3K) \cdot A_0 \cdot L(-3-3K) + (1-3K) \cdot A_3 \cdot L(-3K) = 0\ ; \tag{21}$$

$$(-2-3K) \cdot \mathrm{A}_0 + (1-3K) \cdot A_3 \cdot L^3 = 0\ ; \tag{22}$$

$$L = \sqrt[3]{\left(\frac{2+3K}{1-3K}\right)\left(\frac{A_0}{A_3}\right)}. \tag{23}$$

Conclusion: the exact value of K can make a big difference also in the case when the size-independent part of cost (A_0) competes with the material cost ($A_3 \cdot L^3$).

V. Evaluation of the Power Law Coefficient *K* in literature.

In the middle of the sea $A_2 = 0$ and the K factor is very small ($K = 0$), which means that on height and near water the wind speed is equal. This is the only situation when there is no need to measure the local K factor. A simplified calculation will give a paradoxical result – a smaller turbine is better. However, if artificial islands are created, the formula must also include the water depth because the cost of materials depends on it.

In the seashore, land is very expensive, and the K factor is small only when the wind blows from sea to shore. If it blows in the opposite direction, the K factor can be bigger. Energy is in high demand mostly in winter. Though, in the winter period the wind blows mostly from land to sea.

In other words, the K factor should also be measured first, prior planning wind turbines at the seashore. It is a good idea to measure it only in winter when need of energy is higher.

In [11], it is shown experimentally how the sun's activity influences the K factor – it diminishes in the middle of a sunny day and in summer. This means that to make correct measurements of the K, it is better to make them early in the morning or in winter time.

In [12], Stefan Emeis investigates situations in which the power law is a good approximation. He shows that thermal stratification of air can be a reason for the violation of simple stable power law. Further elaboration of the wind profile power law can be found in [13].

Forests absorb the wind and increase the K factor. According to [14], trees in Latvian forests in some areas can be up to 40–60 meters high.

VI. Possible methods for Measuring the Wind at Height.

Wind measurement masts with sensors mounted at various heights can be used. Though, the use of a mast is economically reasonable up to a height of L <10 m. Above this limit, the cost of a mast increases as L^3 or even L^4. It is reasonable to use a 10 m high mast for reference purposes only. The much bigger mast was once used at coastal sites in Malaysia [15].

Other methods should be used to measure the wind at a larger height: quadcopter drones, which rise up to 300 m, can be used instead of big and expensive masts. Still, there are some complications: the drone itself creates strong wind, they are expensive, and it could become necessary to purchase more than one quadcopter for measurements because they can quickly break down or remain on a tree when used in forests; in some areas with uniform wind (like sea shore) a kite can be used for measuring the wind; free helium balloons are traditionally used by Montgolfier pilots to analyse wind. Though, this is not a good idea to measure the wind in this way for the purposes of wind generation projects because balloons can achieve a height of >60 m only in calm weather. If wind is more than 2 m/s, the helium balloon simply flies away before it manages to achieve suitable height. Fastened helium balloons are theoretically discussed by Geoff Stapleton in [16]. Unfortunately, the fastened helium balloons quickly lean down at wind speeds >2 m/s.

In [17], a Doppler lidar system is proposed. Some measurements practically made by mini-SODAR are discussed in [18] or [19]. This acoustic equipment looks very massive and is very difficult for transportation and is difficult to use it in forest.

VII. CONCLUSIONS

The presented mathematical calculations help to understand how the concept of the power law coefficient *K* of local wind profile can help to make a correct decision on the optimal size of wind turbines. Undoubtedly, to make decisions regarding the purchase, real market prices should be considered.

Nowadays, the *K* factor is no measured systematically prior to the construction. It gives a reason to suspect that in many cases the size of wind turbines is much bigger than their optimal size should be.

To make wind power generation profitable, correct local measurements of the wind at a specific height and calculations of the economically optimal size of wind turbines should be performed before the construction of each wind turbine.

The state programme for the scientific study of the wind (from the point of view of wind power generation) is recommended to be initiated in Latvia.

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