doi: 10.7250/tae.2014.002
2014 / 1

Airline and Aircraft Reliability

Maris Hauka[1], Jurijs Paramonovs[2], [1-2] *Institute of Aeronautics, Faculty of Transport and Mechanical Engineering, Riga Technical University*

Abstract **– Development of the inspection programme of fatigue-prone aircraft construction under limitation of airline fatigue failure rate. The highest economical effectiveness of airline under limitation of fatigue failure rate and failure probability is discussed. For computing is used exponential regression, Monte Carlo method, Log Normal distribution, Markov chains and semi-Markov process theory. The minimax approach is offered for processing the results of full-scale fatigue approval test of an airframe. Fatigue crack parameters and numerical examples are given and explained.**

Keywords **– Inspection programme, Markov chains, minimax, reliability, fatigue crack, exponential approximation.**

I. Introduction

The fatigue failure probability (FFP) of fatigue-prone aircraft (AC) and fatigue failure rate (FFR) of airline (AL) are problems of high priority. A lot of papers and books examine these problems and offer possible solutions [1] – [9] where the Markov chains (MC) and semi-Markov process with reward (SMPW) theories [10] – [12] are offered to solve these problems, using exponential approximation of fatigue crack size growth function, (1) where α, Q are parameters of fatigue crack trajectory (PFCT).

$$a(t) = \alpha \exp(Qt) \tag{1}$$

The value α is called the equivalent initial flow size (EIFS). (Note, it is not a real initial flow size; it is only a parameter of exponential approximation of fatigue crack trajectory!) The value Q defines the speed of fatigue crack size growth on a logarithmic scale: $\log(a(t)) = \log\alpha + Qt$. PFCT are random variables. It is supposed that the cumulative distribution function (cdf) of the vector (α, Q) is known, but a certain parameter of this cdf, θ is not known. Estimation of θ and the choice of inspection programme under condition of limitation FFP up to a specified life (AC retirement age), t_{SL}, or limitation of FFR of AL can be achieved using minimax processing of results of observation of some random fatigue cracks during AC type full-scale fatigue approval test. A specific feature of the approval test is a decision to redesign the new AC type if some reliability requirements are not met. In [1], it was assumed that α was some constant. In this paper this assumption is eliminated.

II. Minimax Choice of Inspection Programme

Despite all the simplicity, formula (1) gives us a rather comprehensible result in the interval (t_d, t_c), where t_d is a time when the crack becomes detectable [13-15] ($a(t_d) = a_d$) (2) and t_c is a time when the crack reaches its critical size ($a(t_c) = a_c$) (3) and fatigue failure takes place (see Fig. 1).

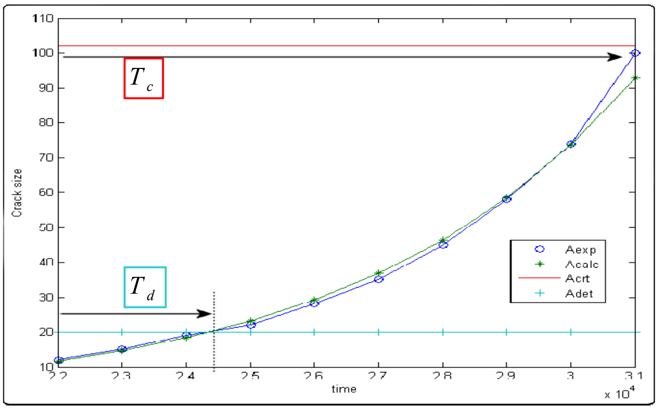


Fig. 1. Exponential approximation of fatigue crack.

$$T_d = (\log a_d - \log\alpha)/Q = C_d/Q \tag{2}$$

$$T_c = (\log a_c - \log\alpha)/Q = C_c/Q. \tag{3}$$

Let us denote $X = \log Q$ and $Y = \log C_c$, where $C_c = \log a_c - \log\alpha$. From the analysis of the fatigue test data it can be assumed that $\log T_c = \log C_c - \log Q$ is distributed normally. It results from the additive property of the normal distribution that can take place if either both $\log C_c$ and $\log Q$ are normally distributed or if one of these components is normally distributed, while the other is constant. Contrary to [1], in this paper we consider the first case: vector $(X,Y) = (\log(Q), \log(C_c))$ has two-dimensional normal distribution with vector-parameter $\theta = (\mu_X, \mu_Y, \sigma_X, \sigma_Y, r)$. It is worth noting that for the case when a_c and a_d are constants, cdf of C_d is completely defined by the distribution of C_c because $C_d = C_c - \delta$, where $\delta = \log(a_c/a_d)$. When θ is known, there are two decisions d_0 and d_1: the aircraft is good enough and the operation of this aircraft type can be allowed (d_0) or the redesign of aircraft should be carried out (d_1). In case of the first decision, vector $\vec{t} = (t_1, ..., t_n)$, where t_i is the time moment of *i*-th inspection, should also be defined. If θ is known the different rules can be offered for the choice of structure of vector $\vec{t}$: 1) every interval between inspections is equal to $t_{SL}/(n+1)$, 2) probability of failure in every interval is equal to $P(T_C < t_{SL})/(n+1)$… In this paper we suppose that (just as in the above-mentioned examples) vector $\vec{t}$ is defined by means of fixed t_{SL} and choice of n.

To substantiate the choice of inspection number, we should know FFP of AC and FFR and gain (GL) of AL as functions of n. For this purpose, the process of operation of AC can be viewed as absorbing MC with $(n+4)$ states. States $E_1, E_2, ..., E_{n+1}$ correspond to AC operation in time intervals $[t_0,t_1),[t_1,t_2),...,[t_n,t_{SL})$, and states E_{n+2}, E_{n+3}, and E_{n+4} are absorbing states: AC is discarded from service when SL is reached or there is a fatigue failure (FF), or fatigue crack detection (CD) takes place (see Fig. 2).

	E_1	E_2	E_3	…	E_{n-1}	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	u_1	0	…	0	0	0	0	q_1	v_1
E_2	0	0	u_2	…	0	0	0	0	q_2	v_2
E_3	0	0	0	…	0	0	0	0	q_3	v_3
…	…	…	…	…	…	…	…	…	…	…
E_{n-1}	0	0	0	…	0	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	…	0	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	…	0	0	0	u_{n+1}	q_{n+1}	v_{n+1}
E_{n+2} (SL)	0	0	0	…	0	0	0	1	0	0
E_{n+3} (FF)	0	0	0	…	0	0	0	0	1	0
E_{n+4} (CD)	0	0	0	…	0	0	0	0	0	1

Fig. 2. Transition probability matrix P_{AC}.

In the transition probability matrix, P_{AC}, for corresponding process of AC operation let the probability of crack detection during the inspection number i be denoted as v_i (6); probability of failure in service time interval $t \in (t_{i-1}, t_i]$ be denoted as q_i (5) and probability of successful transition to the next state as u_i (6). In our model we also assume that an aircraft is discarded from service at t_{SL} even if there are no cracks discovered by inspection at time moment t_{SL}. This inspection at the end of (n+1)-th interval (in state E_{n+1}) does not change reliability but it is carried out in order to know the state of an aircraft (whether there is a fatigue crack or there is no fatigue crack). It can be shown in equations (4-6) where a_i, g_{ai}, b_i, g_{bi}, $\mu_{X/y}$, $\sigma_{X/y}$ is defined in equations (7-12)

$$u_i = P(T_d > t_i \mid T_d > t_{i-1}) = \\ = P(Q < C_d / t_i) / P(Q < C_d / t_{i-1}) = a_i / a_{i-1}, \quad (4)$$

$$q_i = P(t_{i-1} < T_d < T_c < t_i \mid Td > t_{i-1}) = \\ = \begin{cases} 0, & if \quad t_{i-1}C_c / C_d > t_i, \\ b_i / a_{i-1}, & if \quad t_{i-1}C_c / C_d \le t_i, \end{cases} \quad i = 1,...,n+1, \quad (5)$$

$$v_i = 1 - u_i - q_i, \quad (6)$$

where

$$a_i = P(Q < C_d / t_i) = \int_{\ln\delta}^{+\infty} (g_{ai}(y)) d\Phi\left(\frac{y-\mu_Y}{\sigma_Y}\right), \quad (7)$$

$$g_{ai} = P(Q < C_d / t_i) = \\ = \Phi\left(\frac{\left(\log\left(e^y - \delta\right) - \log t_i\right) - \mu_{X/y}}{\sigma_{X/y}}\right), \quad (8)$$

$$b_i = P(C_c / t_i < Q < C_d / t_{i-1}), \quad (9)$$

$$= P\left(\log C_c - \log t_i \le \log Q < \log(C_c - \delta) - \log t_{i-1}\right),$$

$$= \int_{\ln\delta}^{+\infty} (g_{bi}(y)) d\Phi\left(\frac{y-\mu_Y}{\sigma_Y}\right),$$

$$g_{bi}(y) = \max\begin{pmatrix} 0, \Phi\left(\frac{\left(\log\left(e^y - \delta\right) - \log t_{i-1}\right) - \mu_{X/y}}{\sigma_{X/y}}\right) \\ -\Phi\left(\frac{\left(y - \log t_i\right) - \mu_{X/y}}{\sigma_{X/y}}\right) \end{pmatrix}, \quad (10)$$

$$\mu_{X/y} = \mu_X + r\frac{\sigma_X}{\sigma_Y}(y - \mu_Y), \quad (11)$$

$$\sigma_{X/y} = \sigma_X\sqrt{1-r^2}. \quad (12)$$

These probabilities can also be calculated using the Monte Carlo method (13). Equation (14) can be used for modelling r.v with some coefficient of correlation r where r.v. η_1 and η_2 have the standard normal distribution.

$$Y = \log C_c \sim N(\mu_Y, \sigma_Y^2), X = \log Q \sim N(\mu_X, \sigma_X^2) \quad (13)$$

$$Y = \eta_1\sigma_Y + \mu_Y, \; X = \eta_1\sigma_X r + \eta_2\sigma_X\sqrt{1-r^2} + \mu_X \quad (14)$$

Let us recall that in the matrix, P_{AC}, there are three units in three last lines in a diagonal matrix because states E_{n+2}, E_{n+3}, and E_{n+4} are absorbing states: AC is discarded from service when SL is reached or there is a fatigue failure (FF), or fatigue crack detection (CD) takes place.

In the corresponding matrix for operation process of AL, states E_{n+2}, E_{n+3} and E_{n+4} are not absorbing ones and correspond to return of MC to state E_1 (AL operation returns to the first interval). The other lines of P_{AC} and P_{AL} are the same.

For SMPW version of problem, by using P_{AL} we can obtain the airline gain (15), where $\pi = (\pi_1,...,\pi_{n+4})$ is the vector of stationary probabilities, which is defined by (16)

$$g(n) = \sum_{i=1}^{n+4} \pi_i g_i(n) \quad (15)$$

$$\pi P = \pi, \quad \sum_{i=1}^{n+4} \pi_i = 1 \quad (16)$$

$$g_i(n) = \begin{cases} a_i \cdot u_i + b_i \cdot q_i + c_i \cdot v_i, & i = 1,...n+1, \\ d_i, & i = n+2,...,n+4, \end{cases} \quad (17)$$

AL operation rewards are defined in (17), where a_i is the reward related to a successful transition from one operation interval to the next one and the cost of one inspection; b_i, c_i and d_i are related to the transition to states E_{n+3} (FF), E_{n+4} (CD) and E_1. Let us note that if *a=b=c=1, d=0* time transition to state E_1 equals zero, then $\pi_{tj} = \pi_j g_j(n)/g(n)$ defines the time, which is spent by SMP in state E_j $j=1, ..., n+1$, $L_j g(n)/\pi_j$ defines the mean return time for state E_j.

Specifically, L_{n+3} is the mean time between FF; so $\lambda_F = 1/L_{n+3}$ is FFR. It is also worth mentioning that the same value can be calculated in another way. This value is equal to the ratio of aircraft failure probability, p_F, to the mean life of a new aircraft, $L_1 = g(n)/\pi_1$ (the mean time of renewal of AC (renewal operation of AL in the first interval)).

There are two versions of reliability requirements: A) limitation of FFR of AL; and B) limitation of FFP of AC. We will thoroughly consider case A. If θ is known, we calculate the gain as a function of n, $g(n,\theta)$, and choose number n_g corresponding to the maximum of gain. Then we calculate FFR as a function of n, $\lambda_F(n,\theta)$, and choose n_λ in such a way that for all $n \ge n_\lambda$ function $\lambda_F(n,\theta)$ will be equal to or less than some value λ_{FD} (the "designed" FFR) (18). Finally, we choose inspection number n (19).

$$\begin{aligned} & n_\lambda(\lambda_{FD},\theta) = \\ & = \min\{n : \lambda_F(n,\theta) \le \lambda_{FD}, \text{ for all } n \ge n_\lambda(\lambda_{FD},\theta)\} \end{aligned} \quad (18)$$

$$n = n_{g\lambda}(\lambda_{FD},\theta) = \max(n_g(\theta), n_\lambda(\lambda_{FD},\theta)) \quad (19)$$

However, we do not know θ and we can get only some estimate of this parameter, $\hat{\theta}$. Then, first of all, we should define some part of parameter space Θ_0 in such a way that if $\hat{\theta} \notin \Theta_0$ then redesign of AC should be carried out.

If instead of $n_{g\lambda}(\lambda_{FD},\theta)$ we use $\hat{n}_{g\lambda} = n_{g\lambda}(\lambda_{FD},\hat{\theta})$ then real intensity FFR will be a function of random variable, $\lambda_F(\hat{n}_{g\lambda},\theta)$. Let us define $\lambda_F(\hat{\theta},\lambda_{FD},\Theta_0) = \lambda_F(\hat{n}_{g\lambda},\theta)$ if $\hat{\theta} \in \Theta_0$ and $\lambda_F(\hat{\theta},\lambda_{FD},\Theta_0) = 0$ if $\hat{\theta} \notin \Theta_0$ (service of this type of AC is not allowed). The corresponding expected value of FFR as a function of θ has its maximum because in case of "bad $\hat{\theta}$" we redesign an airframe, but in case of "very good $\hat{\theta}$" we do not need any inspection.

Let us denote by $\lambda^*_{FD}(\Theta_0)$ the solution to (20) (if there is the solution to this equation for specific Θ_0), where w_λ (21) is 'required FFR' defined by specific aviation regulations.

$$\sup_\theta w_\lambda(\theta,\lambda_{FD},\Theta_0) = \lambda^* \quad (20)$$

$$w_\lambda(\theta,\lambda_{FD},\Theta_0) = E\{\lambda_F(\hat{\theta},\lambda_{FD},\Theta_0)\},\ \lambda^* \quad (21)$$

$$\begin{aligned} & w_\lambda(\theta,\lambda_{FD},\Theta_0) = \\ & = \{\int_{\Theta_0} \lambda_F(n_{g\lambda}(\lambda_{FD},x),\theta) * \\ & * dF_{\hat{\theta}}(x \mid \hat{\theta} \in \Theta_0)\} P(\hat{\theta} \in \Theta_0). \end{aligned} \quad (22)$$

$$\begin{aligned} & w_p(\theta,p_{FD},\Theta_0) = \\ & = \sum_{i=1}^{n+1} P(t_{i-1}(\hat{\theta}) \le T_d < T_c < t_i(\hat{\theta}), \hat{\theta} \in \Theta_0)\ , \end{aligned} \quad (23)$$

$$\begin{aligned} & n_p(p_{FD},\theta) = \\ & = \min\{n : p_F(n,\theta) \le p_{FD}, \text{ for all } n \ge n_p(n_{FD},\theta)\}, \end{aligned} \quad (24)$$

If after the approval test we see that $\hat{\theta} \in \Theta_0$ then required inspection number $n = n_{g\lambda}(\lambda^*_{FD},\hat{\theta})$. In a similar way the choice of n can be made for case B. Instead of (22) where $F_{\hat{\theta}}(.)$ is cdf of $\hat{\theta}$, the following p-set function [1] should be used (23), where $t_0 = 0$, $t_{n+1} = t_{SL}$, $t_i(\theta)$, $i = 1,...,\hat{n}_{gp}$,

$\hat{n}_{gp} = \max(n_g, n_p(p_{FD},\hat{\theta}))$, p_{FD} is "designed" as allowed FFP of AC, which is used for the choice of n_p (24), function $p_F(n,\theta)$ defines FFP of AC for specific n and θ.

TABLE I

FATIGUE CRACK PARAMETERS

No.	Crack #	Ln (a0)	Q	X=Ln(Q)	Y=lnCc
1	75	-1.2513	1.86E-04	-8.58976	1.905519
2	92	-1.8768	1.95E-04	-8.54251	1.994482
3	93	-1.2445	1.61E-04	-8.73411	1.904507
4	116	-1.697	2.20E-04	-8.42188	1.96971
5	112	-1.5102	2.07E-04	-8.48279	1.943306
7	77	-2.5329	2.28E-04	-8.38616	2.080003
8	78	-0.6479	1.54E-04	-8.77856	1.81148
10	129	-1.4226	1.57E-04	-8.75926	1.93068
	Average	-1.5229	0.000189	-8.5868804	1.942461
	StdDev	0.5480844	2.9E-05	0.1551287	0.077889
	CORREL r				0.796

III. NUMERICAL EXAMPLE

In Table 2.1 of [1] some a priori information about a fatigue crack growth function is provided. Using this information for a_c =237.8 mm and a_d =20 mm (these value have already been used in [1]) we get the following estimate of parameter $\hat{\theta} = (\hat{\mu}_X, \hat{\mu}_Y, \hat{\sigma}_X, \hat{\sigma}_Y, \hat{r})$ = (-8.587, 1.942, 0.155, 0.0779, 0.796) (see Table I). It is supposed that all inspection intervals are equal. The following definition of components of AL income is used: for all $i = 1,...,n+1$ $a_i = a(n) = a_0(n) + d_{insp} t_{SL}$, where $a_0(n) = a_{01} t_{SL} / (n+1)$ is the reward related to a successful transition from one operation interval to the next one; a_{01} defines the reward of operation in one time unit (one hour or one flight); $d_{insp} t_{SL}$ is the cost of one inspection (a negative value), which is supposed to be proportional to t_{SL} ; $b_i = b_{01} t_{SL}$ is related to FF (a negative value), $c_i = c_{01} a_0(n)$ is the reward related to transitions from any state $E_1,...,E_{n+1}$ to state E_{n+4} (it is supposed to be proportional to a_0 because it is part of a_0); $d_i = d_{01} t_{SL}$ is a negative reward, the absolute value of which is the cost of new aircraft acquisition after events SL, FF or CD and the transition to E_1 . In the numerical example we have used the following values (see Table II).

TABLE II

ECONOMIC PARAMETERS

Symbol	Event	Values
b_{01}	Transition to state E_{n+3} (FF)	-3
d_{insp}	Inspection cost	- 0.05
a_{01}	Reward related to a successful transition from one operation interval to the next one	1
c_{01}	Transition to state E_{n+4} (CD)	0.05
d_{01}	Transition to state E_1	- 0.3

Set Θ_0 is defined in the following way: the redesign of the AC type should be carried out if an estimate of mean AC life is small ($T_c < t_{SL}$) or a speed of fatigue crack growth is large ($\log Q > \hat{\mu}_X + \sigma_X$). For λ_{FD} =0.0000001 in Fig. 3a the results of calculation of $w_\lambda(\theta, \lambda_{FD}, \Theta_0)$ and corresponding Fig. 3b (25), where $t_0 = 0$, $t_{n+1} = t_{SL}$, $t_i(\theta)$, $i = 1,...,\hat{n}_{g\lambda}$, $\hat{n}_{g\lambda} = n_{g\lambda}(\lambda_{FD}, \hat{\theta})$, as a function of μ_X for ($\mu_{Y1},..., \mu_{Y5}$)= (1.55, 1.75, 1.94, 2.14, 2.33) in the vicinity of its maximum are shown.

$$w_{p\lambda}(\theta, \lambda_{FD}, \Theta_0) = \\ = \sum_{i=1}^{n+1} P(t_{i-1}(\hat{\theta}) \le T_d < T_c < t_i(\hat{\theta}), \hat{\theta} \in \Theta_0) \qquad (25)$$

It is supposed that vector (σ_X, σ_Y, r) is the same for different vectors (μ_X, μ_Y) and it is equal to the test estimate (0.155128668, 0.0778895, 0.796437). (Let us recall that $(\mu_Y - \mu_X)$ is equal to $E(\log(T_C))$).

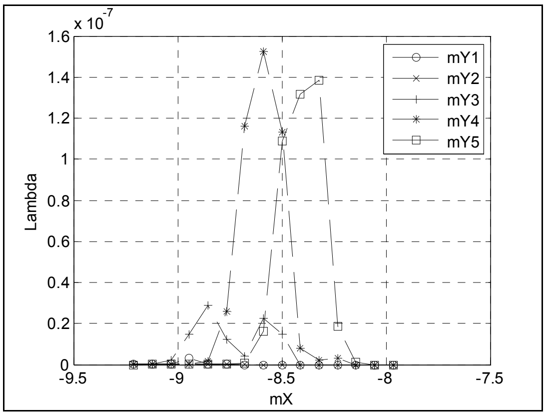


Fig. 3a. $w_\lambda(\theta, \lambda_{FD}, \Theta_0)$ as a function of μ_X .

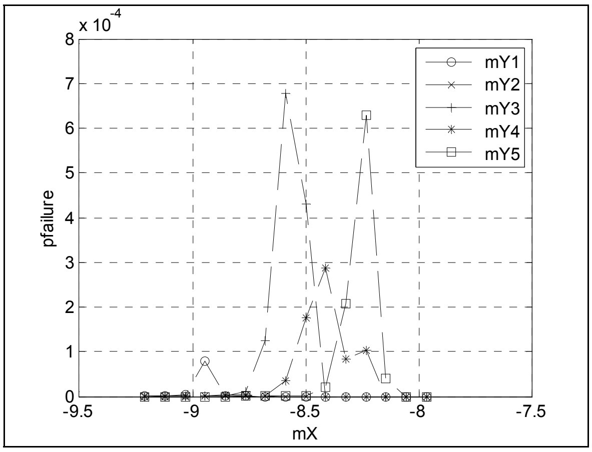


Fig. 3b. $w_\lambda(\theta, \lambda_{FD}, \Theta_0)$ as a function of μ_X .

Maximum value of $w_\lambda(\theta, \lambda_{FD}, \Theta_0)$ is equal to 1.523*10^{-7}. Maximum value of $w_{p\lambda}(\theta, \lambda_{FD}, \Theta_0)$ is equal to 0.000677. Let us suppose that these values satisfy the requirements: required FFR of AL and required FFP of AC. Now let us suppose that in a real test we have got $\hat{\mu}_X$ = -8.5885, $\hat{\mu}_Y$ =1.942460769 (see Fig. 4). (These values have already been considered in [1]). After calculating FFR and airline gain (see Fig. 5), for these specific parameters we find a required number of inspections: $n = \max(n_g, n_\lambda) = \max(3,4) = 4$. It appears that the influence of scatter of EIFS, α, is very significant. After similar calculations of $w_\lambda(\theta, \lambda_{FD}, \Theta_0)$ as a function of μ_X for $\sigma_Y = 0.00001$, $r = 0$ we get its maximum value equal to $1.87 * 10^{-8}$. It is nearly 10 times lower than in the previous case (when σ_Y =0.0778895). Therefore, it is very important to take into account the scatter of EIFS, α.

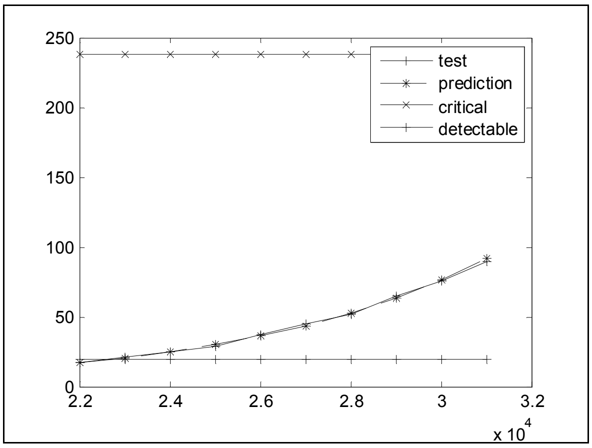


Fig. 4. Example of fatigue crack size as a function of flight number.

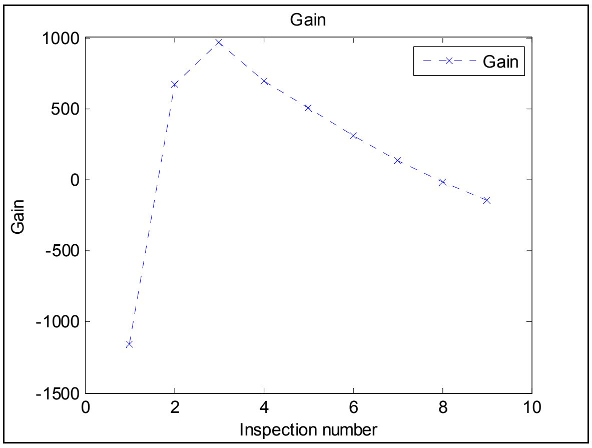


Fig. 5a. Airline gain as a function of inspection number for specific $\hat{\mu}_X$ = -8.5885, $\hat{\mu}_Y$ =1.942460769.

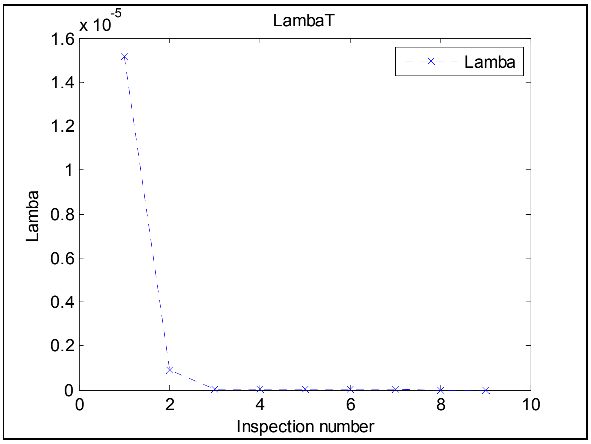


Fig. 5b. FFR as a function of inspection number for specific $\hat{\mu}_X$ = -8.5885, $\hat{\mu}_Y$ =1.942460769.

IV. CONCLUSIONS

Here is shown as the full-scale fatigue approval test of an airframe can be used for aircraft inspection program develop, based not only on limitation of fatigue failure rate, but extends to the economic analysis of results the best likelihood is achieved using minmax method.

ACKNOWLEDGEMENTS

EIROPAS REĢIONĀLĀS ATTĪSTĪBAS FONDS
EIROPAS SAVIENĪBA
IEGULDĪJUMS TAVĀ NĀKOTNĒ

The research has been supported by the European Regional Development Fund within the framework of project “Development of an Unmanned Aircraft System and Creation of the Industrial Prototypes of Unmanned Aerial Vehicles for Performing the Tasks of Latvian National Economy” No.2010/0256/2DP/2.1.1.1.0/10/APIA/VIAA/070

REFERENCES

[1] Y. Paramonov, A. Kuznetsov and M. Kleinhofs, *Reliabilty of fatigue-prone airframes and composite materials,* Riga, RTU Publishing house, 2011.

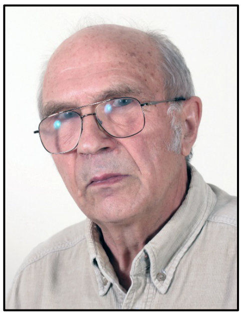
[2] M. Hauka and Y. Paramonovs, *Inspection Program Development Using Minimax Method: 11th International Conference “Reliability and Statistics in Transportation and Communication*”, pp. 105–114, October 19–22, 2011 Riga, Latvia.

[3] M. Hauka and Y. Paramonovs, Inspection Program for the Case of Two Random Parameters of Fatigue Crack: 12th International Conference “Reliability and Statistics in Transportation and Communication”, pp. 78–84, October 17–20, Riga, Latvia.

[4] J. R. Mohanty, B. B. Vermaa and P. K. Ray, “Prediction of fatigue crack growth and residual life using an exponential model: Part I (constant amplitude loading),” International Journal of Fatigue, vol.31, issue 3, pp. 418-424, March 2009. http://dx.doi.org/10.1016/j.ijfatigue.2008.07.015

[5] J. R. Mohanty, B. B. Verma and P. K. Ray, “Prediction of fatigue crack growth and residual life using an exponential model: Part II (mode-I overload induced retardation),” International Journal of Fatigue, vol.31, issue 3, pp. 425-432, March 2009. http://dx.doi.org/10.1016/j.ijfatigue.2008.07.018
[6] K. Sobczyk, “Modelling of random fatigue crack growth,” Journal of Engineering Fracture Mechanics, vol. 24, issue 4, pp. 609-623, 1986. http://dx.doi.org/10.1016/0013-7944(86)90234-1
[7] R. De Silvar, *Exponential model of fatigue crack: In Proceeding of the International workshop "Aviation Reliability AvR 2000*, pp. 45-48, May 9-10, 2000, Riga, RAU, 2000.
[8] B. F. Jr. Spencer and J. Tang, “Markov process model for fatigue crack growth,” Journal of Engineering Mechanics, vol. 114, issue 12, pp. 2134-2157, 1988.
[9] J. C. Newman, J. W Shaw, B. S. Annigeri and B. M. Ziegler, “Fatigue and Crack Growth in 7050-T7451 Aluminium Alloy Under Constant and Variable-Amplitude Loading,” Journal of Engineering for Gas Turbines Power, vol. 135(2), January 2013. http://dx.doi.org/10.1115/1.4007755
[10] R. A. Howard, *Dynamics programming and Markov Processes*. New York-London: Wiley, 1960.
[11] J. R. Norris, *Markov Chains*. Cambridge: Cambridge University Press, 1997. http://dx.doi.org/10.1017/CBO9780511810633
[12] F. Kachapova, “Representing Markov chains with transition diagrams,” Journal of Mathematics & Statistics. vol. 9, issue 3, pp. 149-154, 2013. http://dx.doi.org/10.3844/jmssp.2013.149.154
[13] R. B. Thompson, “A Unified Approach to the Model-Assisted Determination of Probability of Detection,” Materials Evaluation, vol. 66, issue 6, pp. 667-673, June 2008.
[14] J. S. Knopp et al. “Investigation of a Model-Assisted Approach to Probability of Detection Evaluation,” In: D. O. Thompson and d. E. Chimenti (eds.), Review of Progress in Quantitative Nondestructive Evaluation, vol. 26, pp. 1775-1782, New York, USA: American Institute of Physics, 2007.
[15] C. A. Harding, G. R. Hugo and S. J. Bowles, “Application of Model-Assisted POD Using a Transfer Function Approach,” In: In: D. O. Thompson and d. E. Chimenti (eds.), Review of Progress in Quantitative Nondestructive Evaluation, vol. 28, pp. 1792-1799, New York, USA: American Institute of Physics, 2009.

Maris Hauka in 2009 graduated from Riga Aviation University (former Riga Civil Aviation Engineering Institute), Faculty of Transport and Mechanical Engineering and received a degree of Master of Engineering Science.
Present position: Lecturer and PhD candidate at the Institute of Aeronautics, Riga Technical University.
His fields of research: reliability of technical systems.
Institute of Aeronautics, Faculty of Transport and Mechanical Engineering, Riga Technical University, Lomonosova 1A, k-1, Riga, LV–1019, Latvia.
Phone: +371 67089990
E-mail: Maris.Hauka@rtu.lv



Jurijs Paramonovs. in 1993 was awarded a degree of Hab.Dr.eng by Riga Aviation University and the Latvian Academy of Sciences. In 1974 he was awarded a Doctoral Degree in Technical Cybernetics by the Latvian Academy of Sciences. In 1965 he received a degree of Doctor of Engineering Science from Riga Civil Aviation Engineering Institute.
Present Position: Professor of Aircraft Theory and Structure Department at the Aeronautical Institute of Riga Technical University.
Member of professional societies: American Statistical Association, Research Board of Advisors of the American Biographical Institute. He has participated in 17 international conferences and has written 145 publications, including 9 monographs and textbooks.
His fields of research: reliability of technical systems; mathematical statistics; loads, structure and strength analysis of transport vehicle; information technology application (development of automated control systems).
Institute of Aeronautics, Faculty of Transport and Mechanical Engineering, Riga Technical University, Lomonosova 1A, k-1, Riga, LV–1019, Latvia.
Phone: +371 67089990
E-mail: Jurijs.Paramonovs@rtu.lv