The Use of Skin Effect for Measurement of Thickness of Sputtered Protective-Coatings

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Abstract – **This paper deals with the development of a mathematical model that describes the skin effect in cylindrical conductors with protective coating. The model allows calculating the distribution of current density and magnetic field intensity, as well as determining electrical impedance, resistance, reactance and current in separate layers. It is established that electrical impedance at constant frequencies decreases linearly, if thickness of a protective coating grows. This fact can be used for electrical measurement of thickness of a protective coating.**

Keywords – **Measurement, skin effect, sputtered protective-coating, thickness.**

I. Introduction

The development and implementation of technologies of sputtered protective coatings require control over thickness of these coatings. One of the non-destructive and easily automatable methods that allows measuring thickness of an electrically conductive coating may be associated with the use of skin effect [1]. Let us use the skin effect theory (discussed in many literature sources, e.g., [2] - [6]) to determine in which situations this effect can be used for the measurement of thickness of a protective coating. In work [7] a calculation method of frequency–dependent resistance and internal inductance of a solid cylindrical conductor is developed. Skin effect reduces the volume of sample and changes its electrical resistance [8]. Modeling of skin effect for solving specific engineering problems is viewed in many authors' publications, for example, in [9] - [16].

The purpose of this paper is to develop a mathematical model that describes the skin effect in cylindrical conductors with a protective coating. It would allow assessing necessary conditions and developing a method for electrical measurement of thickness of a protective coating.

II. Mathematical Model

Conductive coating with thickness Δ is sputtered on a conductive cylindrical element with inside/ outside diameter d_1/d_2 (see Fig. 1).

High-frequency alternating current I flows through a cylindrical element in the z-axis direction. In this situation skin effect is observed; high-frequency current flows only through a thin layer of cylindrical conductor's outside diameter. As $\Delta << d_2$, the problem of skin effect can be solved for a one-dimensional case. Cylindrical conductor with sputtered protective coating will be changed to a rectangle conductor with height πd_2 and width b. Alternating current I (in the z-axis direction) flows through it. There is no magnetic field in internal air space ($d<d_1$) of cylindrical body, because

$\oint \vec{H} d\vec{s} = 0$, therefore this space can be not viewed. As a basis for the development of this method, a calculation example from [2] will be used, where on surface $x=0$ the magnetic field intensity $H=0$. Thereby, it remains to observe only the area of conductor, which has coordinates $0 \leq x \leq a$ (let us mark it $\{i\}$) and the current conductive protective coating layer (e.g., TiN with Al) with coordinates $a \leq x \leq b$ (let us mark it $\{m\}$). For these areas one dimensional skin equation is valid with directed vector potential directed along the z-axis $\vec{A} = \vec{e}_z A$.

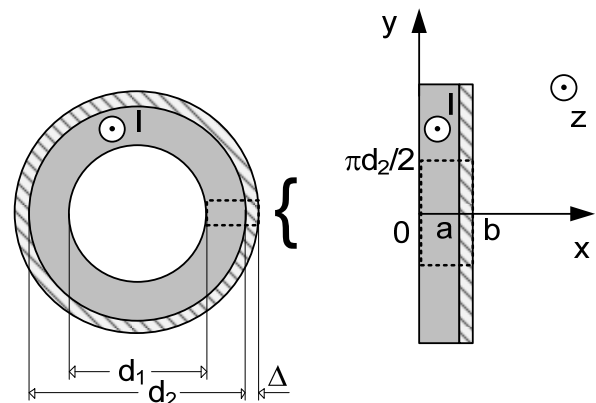$$A_{i/m} = \frac{d^2 A_{i/m}}{dx^2} = a_{i/m}^2 A_{i/m} \qquad (1)$$

Fig. 1. Geometry of the model.

For the description of the field, vector potential $\vec{A}(x)$ with $rot(\vec{A}) = \vec{B}$ is used, where $\alpha = \sqrt{j\omega k\mu} = \frac{1+j}{\delta}$ – the skin constant, $\delta = \sqrt{\frac{2}{\omega k\mu}}$ – the skin depth, ω – the angular frequency of current, k –the specific electrical conductivity, μ – the absolute magnetic permeability, j – the imaginary unit, B – the magnetic flux density.

General solutions to (1) for areas $\{i\}$ and $\{m\}$ are searched as follows:

$$\begin{cases} A_i(x) = C_i \cosh(a_i x) \\ A_m(x) = C_m \cosh(a_m x) + D_m \sinh(a_m x) \end{cases} \qquad (2)$$

In this case, the vector potential does not depend on coordinates y and z ($dA/dy=dA/dz=0$); current flows in the z-axis direction. It means that the magnetic field intensity has y-axis direction $\vec{H}(x) = \vec{e}_y H(x)$. Intensity can be calculated for areas $\{i\}$ and $\{m\}$ using (2) and well-known relevance of electrodynamics $\vec{B} = \mu \vec{e}_y H(x) = rot(\vec{e}_z A) = -\vec{e}_y \frac{dA}{dx}$:

$$\begin{cases} H_i(x) = \frac{\alpha_i}{\mu_i} C_i \sinh(a_i x) \\ H_m(x) = \frac{\alpha_m}{\mu_m} C_m \sinh(a_m x) + \frac{\alpha_m}{\mu_m} D_m \cosh(a_m x) \end{cases} \quad (3)$$

Equation systems (2) and (3) have 3 unknown constants C_i, C_m, D_m. They can be determined using these conditions:

a) condition of continuity of vector potential $A_i(a)=A_m(a)$;
b) condition of continuity of magnetic field intensity $H_i(a)=H_m(a)$;
c) the magnetic field intensity vector integral by closed contour (see a dotted rectangle in Fig. 1) is equal to a sum of covered currents $\oint \vec{H} d\vec{s} = I$. After integration by this contour it is possible to obtain $H_m(b)\pi d_2=I$.

Using these conditions and (2) and (3) it is possible to obtain:

from a) and (2)

$$C_i \cosh(a_i a) = C_m \cosh(a_m a) + D_m \sin(a_m a)$$

from b) and (3)

$$\frac{\alpha_i}{\mu_i} C_i \sinh(a_i a) = \frac{\alpha_m}{\mu_m} C_m \sinh(a_m a) + \frac{\alpha_m}{\mu_m} D_m \cosh(a_m a)$$

from c) and (3)

$$-\frac{\alpha_m}{\mu_m} C_m \sinh(a_m b) - \frac{\alpha_m}{\mu_m} D_m \cosh(a_m b)\pi d_2 = I$$

These equations are written in a matrix form

$$\begin{pmatrix} \cosh(a_i a) & -\cosh(a_m a) & -\sin(a_m a) \\ \frac{\alpha_i}{\mu_i}\sinh(a_i a) & -\frac{\alpha_m}{\mu_m}\sinh(a_m a) & -\frac{\alpha_m}{\mu_m}\cosh(a_m a) \\ 0 & -\frac{\alpha_m}{\mu_m}\sinh(a_m b) & -\frac{\alpha_m}{\mu_m}\cosh(a_m b) \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} C_i \\ C_m \\ D_m \end{pmatrix} = \begin{pmatrix} o \\ 0 \\ \frac{I}{\pi d_2} \end{pmatrix} \quad (4)$$

Unknown constants C_i, C_m and D_m can be calculated numerically. Therefore vector potential (2) and magnetic field intensity (3) in both electrically conductive layers {i} and {m} are determined.

Alternating current density in layers {i} and {m} is calculated using equations of electrodynamics $\vec{J} = k\vec{E}$ and $\vec{E} = -j\omega\vec{A}$ (where E – the electric field):

$$\begin{cases} J_i(x) = -j\omega k_i\, C_i \cosh(a_i x) \\ J_m(x) = -j\omega k_m(C_m \cosh(a_m x) + D_m \sinh(a_m x)) \end{cases} \quad (5)$$

At high frequencies, if calculations are performed throughout the thickness $(d_2-d_1)/2$ of layer {i}, there are problems with numerical solving of (4). Therefore it is assumed:
if $(d_2-d_1)/2>12\delta_i$, then $a=12\delta_i$ – the current in layer {i} flows only near the external surface;

if $(d_2-d_1)/2\le12\delta_i$, then $a=(d_2-d_1)/2$ – the current flows throughout the thickness of layer {i}.

Values of current density in (5) are complex numbers, which consist of real part $J_{Re}(x)$ and imaginary part $J_{Im}(x)$.

In practice, the root mean square (RMS) of current density $JB(x)$ is significant that can be calculated for layers {i} and {m} as follows:

$$JB_{\frac{i}{m}}(x) = \frac{J_{max}(x)}{\sqrt{2}} = \frac{\sqrt{J_{Re}(x)^2+J_{Im}(x)^2}}{\sqrt{2}}, \quad (6)$$

where $J_{max}(x)$ – the AC current density amplitude at a point with coordinate x.

Knowing $J_i(x)$ and $J_m(x)$, it is possible to calculate currents, which flow in layers {i} and {m}:

$$\begin{cases} I_i = \frac{\pi d_2}{\sqrt{2}} = \left|\int_0^a J_i(x)dx\right| \\ I_m = \frac{\pi d_2}{\sqrt{2}} = \left|\int_0^b J_m(x)dx\right| \end{cases} \quad (7)$$

This equation system can be used for checking the numerical calculations. If $I_i+I_m=I$, then calculation is true.

By means of Poynting vector, it is possible to determine inside electrical impedance

$$Z = -I^2 \oiint_A \left(\vec{E} \times \vec{H}\right)d\vec{A}, \quad (8)$$

where E – the electric field, H – the magnetic field intensity on conductor's surface A.

As in the given situation, electric field vector E is directed by the z-axis, magnetic field intensity vector H is directed by the y-axis, as well as on the conductor internal surface $H(0)=0$ and on the conductor external surface $H(b)\neq0$, then

$$Z = \frac{\pi d_2 l}{I^2 k_m}J_m(b)H_m(b) \quad (9)$$

where l – the length of conductor. Impedance of layers {i} and {m}:

$$\begin{cases} Z_i = \frac{\pi d_2 l}{I_i^2 k_i}J_i(a)H_i(a) \\ Z_m = \frac{\pi d_2 l}{I_m^2 k_m}J_m(b)H_m(b) - J_m(a)H_m(a) \end{cases} \quad (10)$$

As both layers of conductor are connected in parallel circuits, then:

$$\frac{1}{Z} = \frac{1}{Z_i} + \frac{1}{Z_m} \quad (11)$$

Z is a complex value, whose real part characterises resistance R, but imaginary part – reactance X.

III. RESULTS AND DISCUSSION

Calculations for a steel tube of length l=153 mm and inside/outside diameter d_1/d_2= 6/8 mm were performed using Mathcad software. A protective coating with thickness Δ is sputtered on the tube. In the calculations such material constants were used: specific electrical conductivity of steel

$\kappa_i=(1,01\pm0,32)\cdot10^6$ *S/m*; relative magnetic permeability of steel $\mu_i=27,9\pm2,0$; specific electrical conductivity of Al-TiN coating $\kappa_m=(0,43\pm0,11)\cdot10^6$ *S/m*, relative magnetic permeability $\mu_m=1$. [1]

An example for the distribution of current density $|J(x)|/\sqrt{2}$ (RMS) in the tube with a protective coating is given in Fig. 2 and Fig. 3, where $|J(x)|$ – the amplitude of current density, coordinate *x* is radially directed; the distribution of magnetic field intensity (amplitude) *H(x)* is depicted – in Fig. 4 and Fig. 5; AC frequency *f=100 MHz*; the total current in conductor *I=33,27 mA*; thickness of a protective coating $\Delta=1,6\mu m$. Units of measurement: *[x]=1 m*; *[J(x)]=1 A/m²*; *[H(x)]=1 A/m*.
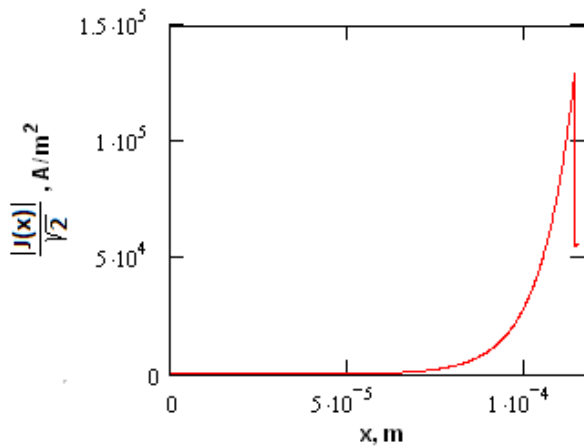
Fig. 2. Distribution of current density in the tube with a protective coating at 100 MHz.
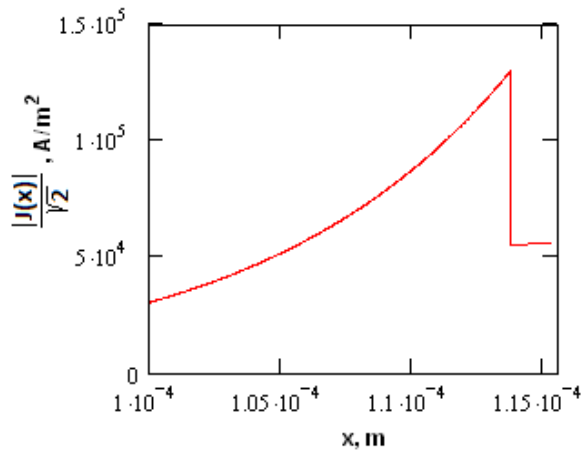

Fig. 3. Distribution of current density in the tube with a protective coating at 100 MHz (expanded scale).
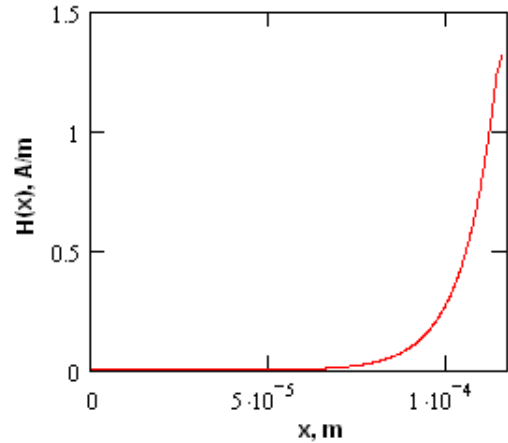

Fig. 4. Distribution of magnetic field intensity in the tube with a protective coating at 100 MHz.
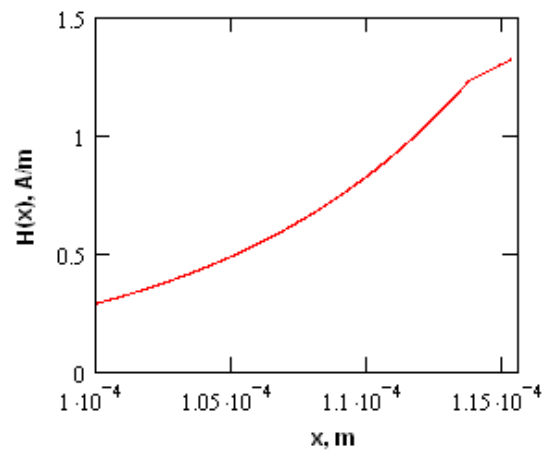

Fig. 5. Distribution of magnetic field intensity in the tube with a protective coating at 100 MHz (expanded scale).
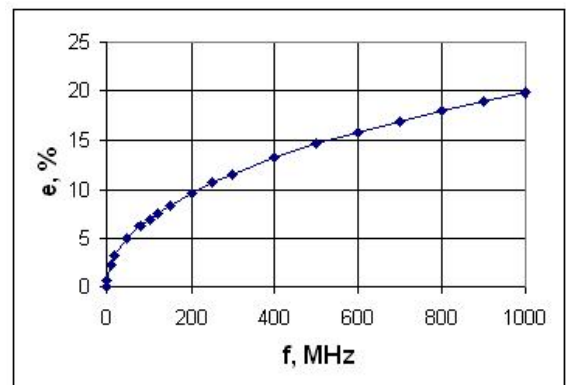

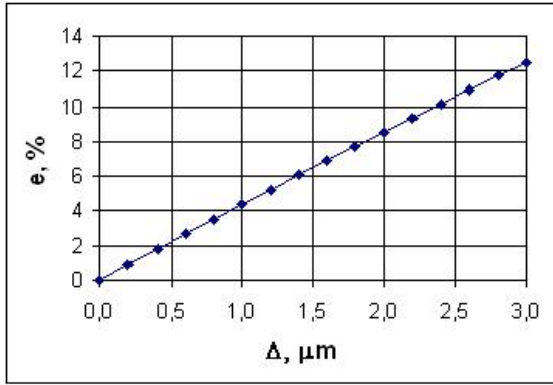Fig. 6. Part of current that flows through a protective coating (thickness of 1.6 μm) depending on AC frequency.

Fig. 7. Part of current that flows through a protective coating depending on the thickness of a layer at a frequency of 100 MHz.

If $f=1MHz$, then in calculations we obtain $Z=0.064+0.063j$ Ω. Experimentally measured resistance for a given sample $R=0.0643$ Ω [1]. Obviously, the numerical and experimental results fully coincide. It confirms the correctness of the given model.
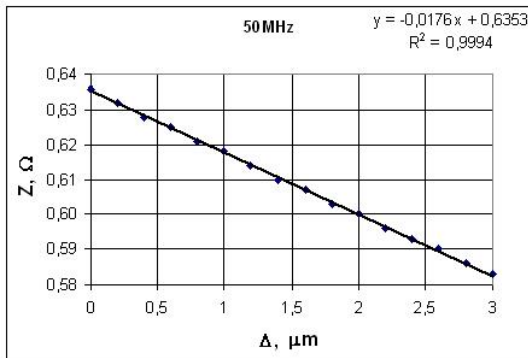

Fig. 8. Electrical impedance depending on the thickness of sputtered layer at 50 MHz.
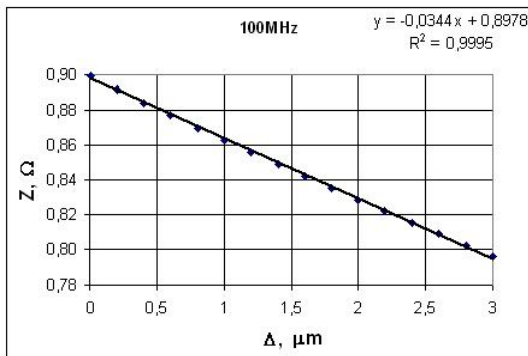

Fig. 9. Electrical impedance depending on the thickness of sputtered layer at 100 MHz.
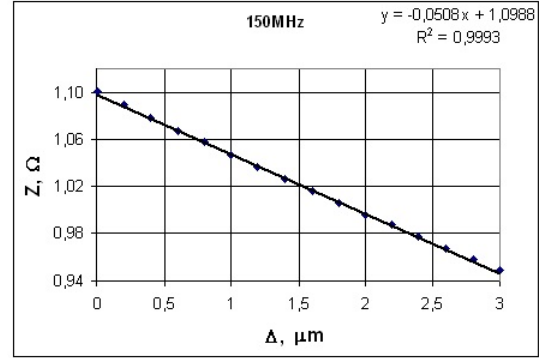

Fig. 10. Electrical impedance depending on the thickness of sputtered layer at 150 MHz.
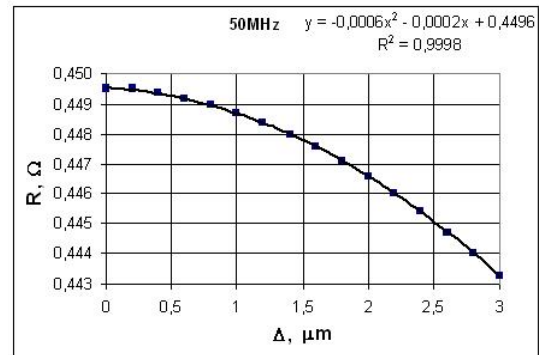

Fig. 11. Electrical resistance depending on the thickness of sputtered layer at 50 MHz.
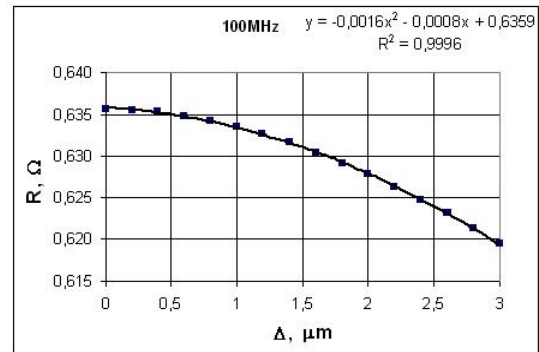

Fig. 12. Electrical resistance depending on thickness of sputtered layer at 100 MHz.
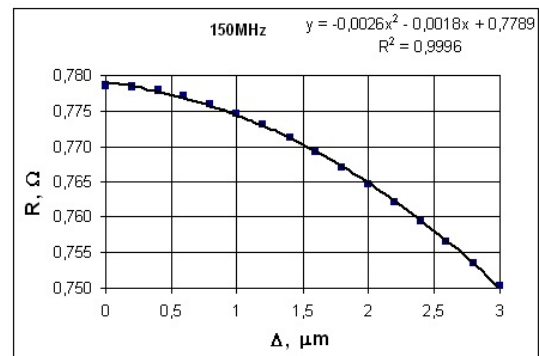

Fig. 13. Electrical resistance depending on the thickness of sputtered layer at 150 MHz.

Let us define value *e*, which characterises part of current that flows in a protective coating:

$$e = \frac{I_m}{I_i + I_m} \qquad (12)$$

Dependence of value *e* on AC frequency at *Δ=1,6μm* is demonstrated in Fig. 6. Dependence of value *e* on the thickness of a protective coating at *f=100 MHz* may be seen in Fig. 7.

The higher value *e*, the larger part of current flows in a protective coating. To measure the thickness of a protective coating electrical gauges can be used with lower accuracy.

Figures 8–13 illustrate the dependence of electrical impedance *Z* and resistance *R* of the tube with a protective coating on the thickness of sputtered layer at AC frequency *f=50 MHz, f=100 MHz, f=150 MHz*.

The higher AC frequency, the faster values *Z* and *R* change depending on the thickness of a protective coating. Consequently, the higher AC frequency, the lower accuracy of electrical gauges is necessary to measure the thickness of a protective coating.

If the thickness of a protective coating at a constant frequency grows, then electrical impedance Z linearly decreases.

IV. CONCLUSIONS

Within the framework of the study a mathematical model has been developed that describes the skin effect in cylindrical conductors with a protective coating. The model can also be used for multilayer sputtered protective coatings – in this case, only the number of equations in systems (2) – (6), (10), (11) grows. The model allows calculating the distribution of current density and distribution of magnetic field intensity in cylindrical conductors with a protective coating, determining electrical impedance, resistance and reactance of separate layers, as well as setting current in each layer at any AC frequency and any thickness of a protective coating.

It is established that electrical impedance at constant frequencies decreases linearly, if the thickness of a protective coating grows. This means it is possible to use the skin effect for the measurement of thickness of a protective coating: at constant frequency it is necessary to measure the electrical impedance of cylindrical conductor without a protective coating and impedance of the same conductor with a protective coating, whose thickness of coating is known; then it is necessary to construct characteristic *Z=Z(Δ)* that in a given situation is a straight line; using this characteristic and measuring electrical impedance for similar conductors with sputtered protective coating, it is possible to determine the thickness of protective coating Δ.

The model allows evaluating the necessary accuracy of electrical gauges (e.g., LCR meter, voltmeter, and ammeter) to measure thickness of a protective coating. Varying the thickness of a protective coating, changes in electrical parameters of cylindrical conductor become more quintessential, if AC with higher frequency is used; it means that at higher frequencies it is possible to use electrical gauges with lower accuracy for the measurement of thickness of a protective coating.

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